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Rainy and Namakan Hydrologic Response Model Documentation



By: Aaron F. Thompson, P.Eng.
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Contact Information for the Author:

Aaron F. Thompson, P.Eng.
Environment Canada
867 Lakeshore Road, Burlington, Ontario, L7R 4A6
Email: Aaron.Thompson@ec.gc.ca
Telephone: 905-336-4959

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1. Introduction

This report documents the development of a hydrologic response (hydrologic routing) model for Rainy Lake and the Namakan Chain of Lakes located along the border of Northwestern Ontario in Canada and Minnesota in the United States (Figure 1). The Namakan Chain of Lakes includes Little Vermilion, Crane, Sand Point, Kabetogama, and Namakan Lakes that ordinarily stand at the same elevation. A hydrologic response model was developed to provide two time series of water levels and outflows for Rainy Lake and the Namakan Chain of Lakes to be used in the further studies in preparation for the International Joint Commission's (IJC) review of the Rainy and Namakan Lakes 2000 Rule Curves. The time series represent the water levels and outflows that would have occurred in the Lakes for the period from 1950-2014, if the 1970 and 2000 Rule Curves regulation strategies were strictly followed. The period from 1970 to 2012 was used to develop and test the model and afterwards the model was extended to cover the period from 1950-2012. Later in 2015, the model was extended to cover the years of 2013 and 2014. The hydrologic response model was developed using Microsoft Excel software and calculates quarter-monthly water levels for the lakes over the entire time period.

The report is organized as follows. Section two presents a brief background to explain why the model is needed and the objectives of the development of the model for the IJC Rainy and Namakan Lake Rule Curves Review. Section three provides a description of the 1970 and 2000 Rule Curves for Rainy and Namakan Lakes followed by descriptions of the types and sources of model inputs, the simplifying assumptions required to develop the model, the basic mathematical equations and decision making processes used by the model. Section three also describes how the model calculates the outflows from Rainy and Namakan Lakes. The development of an algorithm to calculate the outflows from the lakes that is both consistent with the regulation rules and also replicates the historic water levels of Rainy and Namakan Lakes is difficult for a number of reasons that will be summarized near the end of this report. The outflows and water levels calculated by the model reflect this challenge and therefore do not exactly replicate the observed historic values. However, the accuracy of the time series of water levels and outflows are likely sufficient for the future evaluations of the Rule Curves Study because they quantify the differences in the water levels in the system that would have occurred for the period of simulation under the two Rule Curves strategies. Section four of the report evaluates that performance of the model comparing the computed water levels and outflows to the recorded values both graphically and statistically. Finally, section five presents the time series of water levels computed by the model for the period from 1950-2014 under both the 1970 and 2000 Rule Curves management strategies. It also presents monthly histograms of water levels under both Rule Curves strategies and stage-frequency curves that may be useful to the subsequent investigators of the IJC 2015 Rule Curves review.

The model's computation algorithms have been reviewed technically by engineers at the St. Paul District of the U.S. Army Corps of Engineers and the Lake of the Woods Secretariat in Ottawa. The model and the report have also been through an external peer review process conducted by the International Joint Commission. The comments and suggestions from all reviewers have been incorporated into this final report.

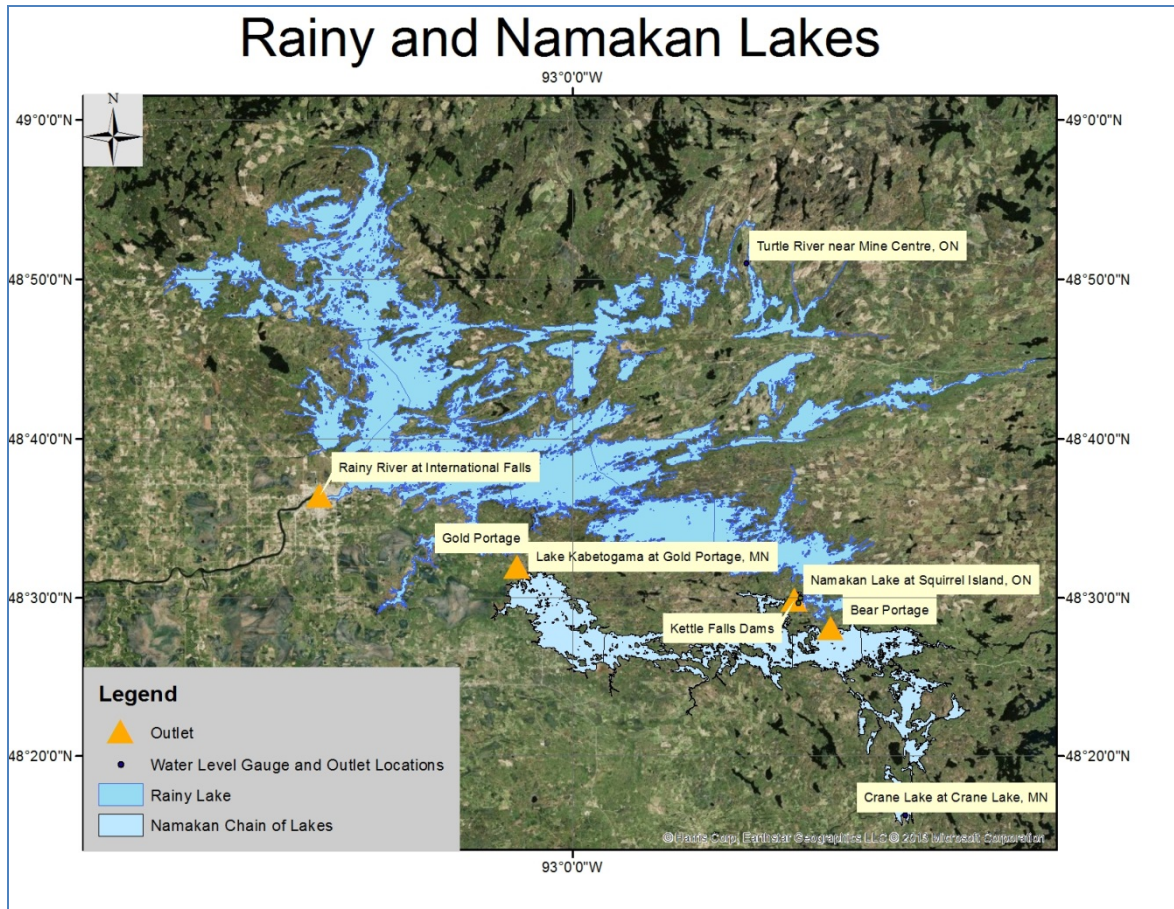


Figure 1 - Rainy Lake and Namakan Lakes Study Area

2. Background and Objectives

In 2001 the International Joint Commission (IJC) issued an Order prescribing the method of regulating the levels of the boundary waters of Rainy and Namakan lakes, consolidating and replacing a number of previous orders and supplementary orders (International Joint Commission 2001). This “Consolidated Order” was effective on February 28, 2001, and contained the following provision: “This order shall be subject to review 15 years following adoption of the Commission's Supplementary Order of 5 January 2000, or as otherwise determined by the Commission. The review shall, at a minimum, consider monitoring information collected by natural resource management agencies and others during the interim that may indicate the effect of the changes contained in the Supplementary Order of January 5, 2000.”

In 2007, the IJC formed a Rule Curves Assessment Workgroup to develop a plan of study (POS) in which the Workgroup would prioritize the monitoring and analyses required to review the IJC Order in 2015. Specifically, the POS was written to identify priority studies and describe information/data that remained to be collected, identify what entities might collect the data and perform the studies, and to provide an estimate for the cost to accomplish this work by 2015. The Plan of Study (POS) for the Evaluation of the International Joint Commission (IJC) 2000 Order for Rainy and Namakan Lakes and Rainy River was completed in 2009 (Kallemeyn et al., 2009). The Plan of study recommended the development of a reservoir hydrologic model for Rainy Lake and the Namakan Chain of Lakes that could perform simulations to develop key lake level timing, elevation frequency and elevation-duration relationships under the 1970 and 2000 Rule Curves regulation strategies. The results from this model would later interface with habitat simulation, coastal and other types of models to determine if differences in the water levels resulting from the two regulation strategies were impacting indicator species or other interests in the basin.

A Scope of Work (SOW) was developed by Environment Canada for a study to develop a reservoir hydrologic model for Rainy Lake and the Namakan Reservoir to help evaluate hydrodynamic changes of the reservoirs due to the 2000 Rule Curves. This study, identified in the POS, is number eight of SOWs to be completed as part of the Cooperative Agreement between the USDI-National Park Service Voyageurs National Park (VNP) and the IJC, Agreement No.-1042-100732. The proposal was accepted by the International Joint Commission.

Therefore, the objective of the development of the hydrologic response model was to provide two time series of water levels and outflows for Rainy Lake and the Namakan Chain of Lakes. The time series and statistical attributes of the time series will be used in the further studies in preparation for the International Joint Commission's (IJC) review of the Rainy and Namakan Lakes 2000 Rule Curves. The model was developed using Microsoft Excel spreadsheet software. The model's algorithms are simple to follow and could be modified in the future should simulations of alternative regulation plans or state-of-nature conditions be required.

3. Rainy and Namakan Hydrologic Response Model

3.1. Rainy and Namakan Lake Regulation

Rainy Lake was first regulated in 1909 and Namakan Lake was first regulated in 1914. The lakes are regulated in accordance with rules put in place by the International Joint Commission. The Commission issued Orders of Approval in 1949 specifying the manner and objective for the regulation of Rainy and Namakan Lakes. The Commission issued Supplemental Orders amending how the lakes were regulated in 1957, 1970, and 2000. The supplemental orders issued in 1970 and 2000 are most relevant to this hydrologic response modelling project. The 1970 Supplemental Orders established emergency high and low water levels for Rainy and Namakan Lake to be avoided by regulating the outflows from the dams for each lake. The Order also defined an upper and lower Rule Curves and prescribed minimum outflows for both lakes, with operation between these curves and above minimum outflows at the discretion of the power Companies. The 2000 Supplemental Orders revised the 1970 upper and lower Rule Curves for both lakes, required the power Companies to target the middle portion of the Rule Curves band subject to other direction from the International Rainy Lake Board of Control, and revised the prescribed minimum outflows. The 1970 and 2000 Rule Curves minimums and maximums for Rainy and Namakan Lakes are shown in the appendix. The 1970 and 2000 Rule Curves are combined and shown graphically on Figures 2 and 3. Since the hydrologic model operates on a quarter-month time step interval, quarter-month Rule Curves were created by interpolating the tabular rating curves and extracting the required water levels for the end-of-period quarter months.

In addition to setting outflows to maintain the water levels between the upper and lower Rule Curves according to the 1970 Supplemental Orders or at the middle of the Rule Curves according to the 2000 Supplemental Orders, the regulation plan also specifies actions to deal with high and low flows. When the water level on Rainy Lake reaches the level of 337.9 metres (1108.6 feet) and inflows exceed outflows to the lake, all gates at the International Falls must be opened. When the water level of Namakan Lake reaches 341.1 metres (1119.1 feet) and inflows exceed outflows, all gates and fish ways at both Namakan Lake outlet dams must be removed.

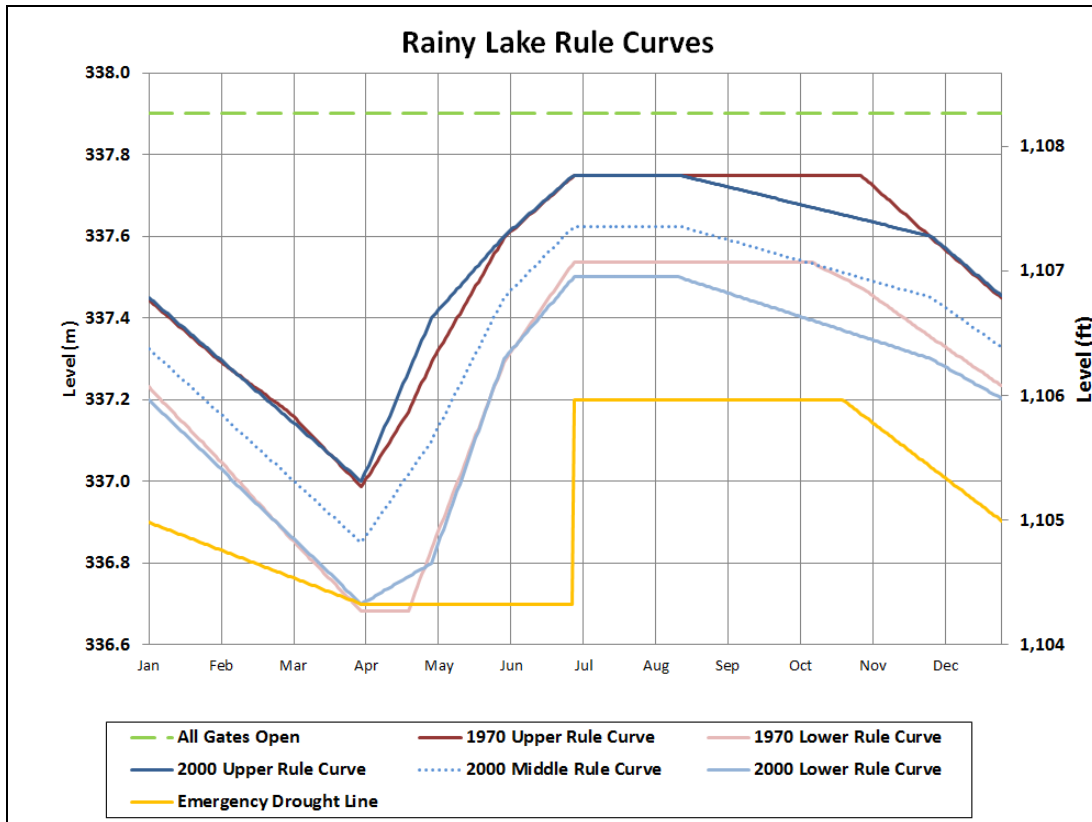


Figure 2 - Rainy Lake 1970 and 2000 Rule Curves

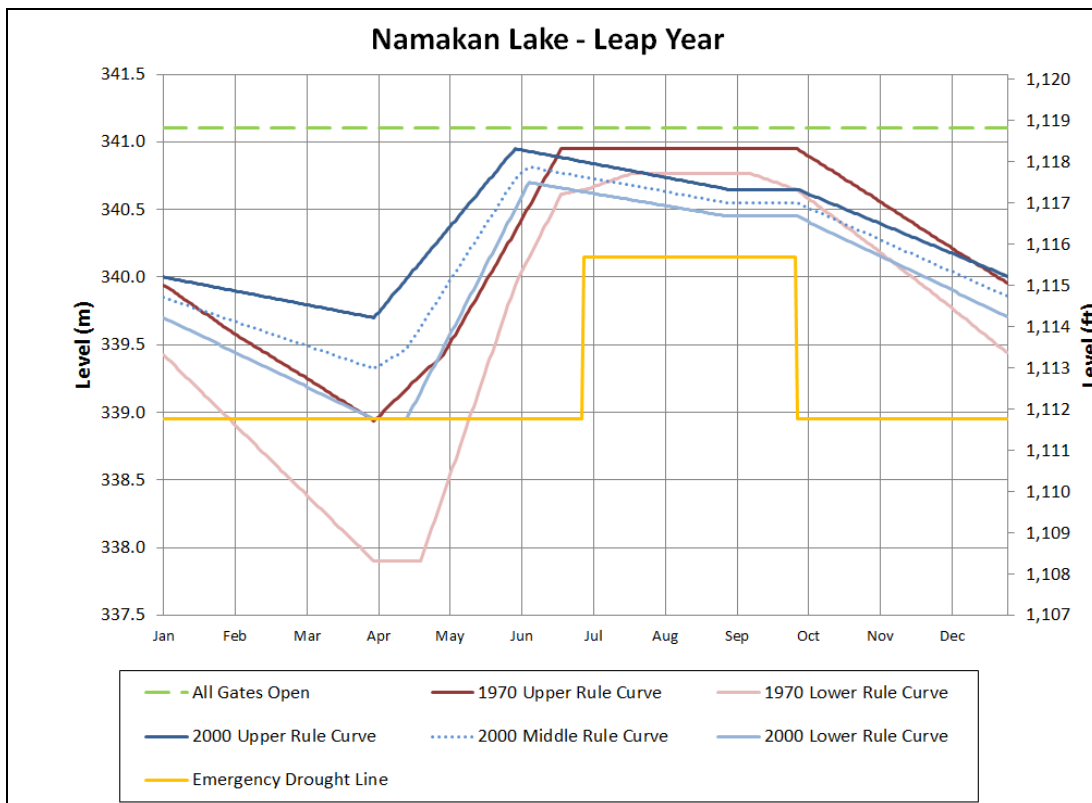


Figure 3 - Namakan Lake 1970 and 2000 Rule Curves

During periods of low water levels under the 1970 Supplemental Orders, when water levels of Namakan Lake fall below the lower Rule Curve, outflows from the two dams at Kettle Falls must be reduced to 28.3 m³/s (1000 ft³/s). When water levels on Rainy Lake fall below the lower Rule Curve, flows must be reduced to the minimum of 113.3 m³/s (4000 ft³/s) between the hours of sunrise and sunset in the months of May to October and 93.4 m³/s (3300 ft³/s) at all other times. The 2000 Supplemental Orders changed the minimum flow requirements for both lakes and established a low water level drought line below which outflows are to be further reduced. Under the 2000 Supplemental Orders when water levels on Namakan Lake fall below the lower Rule Curve, outflows from the two dams at Kettle Falls must be reduced to 30 m³/s (1060 ft³/s) and when water levels fall below the drought line outflows are to be reduced further to 15 m³/s (530 ft³/s). For Rainy Lake, when water levels fall below the lower Rule Curve, outflows must be reduced to 100 m³/s (3530 ft³/s) for all months of the year and when water levels fall below the lower Rule Curve outflows must be reduced to 65 m³/s (2300 ft³/s). These minimum flows are necessary to preserve water quality in the system.

In both the 1970 and 2000 Supplemental Orders if extremely high or low inflows to Namakan Lake or Rainy Lake are anticipated, the International Rainy Lake Board of Control, after obtaining the approval of the IJC, may authorize the levels of Namakan and Rainy Lake be raised temporarily to greater than the maximum or lowered temporarily to less than the minimum elevations of the Rule Curves.

3.2. Model Approach and Computation Interval

The hydrologic response model uses a simple conservation of mass approach (Linsley et al. 1982) that balances inflows, outflows and the change in storage in the lake for each time step:

$$1) \quad I - O = \Delta S \text{ (m}^3\text{)}$$

Where:

I is the inflow to the lake (m³)

O is the outflow from the lake (m³)

ΔS is the change in storage in the lake (m³)

The time steps used for the simulations of this model are one quarter month in length. This time step was selected because Rainy Lake and the Namakan Chain of Lakes have large storage capacities and as a result water levels in the lakes rise and fall slowly. Figures 4 through 6 show the inflows, outflows and water levels for Rainy Lake for year 2001 on daily and quarter-monthly time steps. There are pronounced differences in inflow on daily versus quarter-monthly time steps, less difference in outflow, and essentially no difference in water level. Daily and quarter monthly inflows differ because inflows are dependent on supplies from upstream tributaries that can

vary widely from day to day due to the amount of precipitation received. Lake levels change slowly due to the large storage capacities of the lakes relative to the inflows and thus there is little difference between daily and quarter monthly data. Similarly outflows are largely a function of lake levels and accordingly change slowly over time. The primary products of the hydrologic response model are water level time series so the quarter-monthly time step is appropriate. The length of a quarter month varies throughout the year because of the variability in month length. The International Rainy Lake Board of Control and Lake of the Woods Secretariat has adopted the following standard quarter month lengths. The first quarter month period ends on the 8th of the month, the second ends on the 15th of the month, the third on the 22nd of the month and the fourth end on the month-end day.

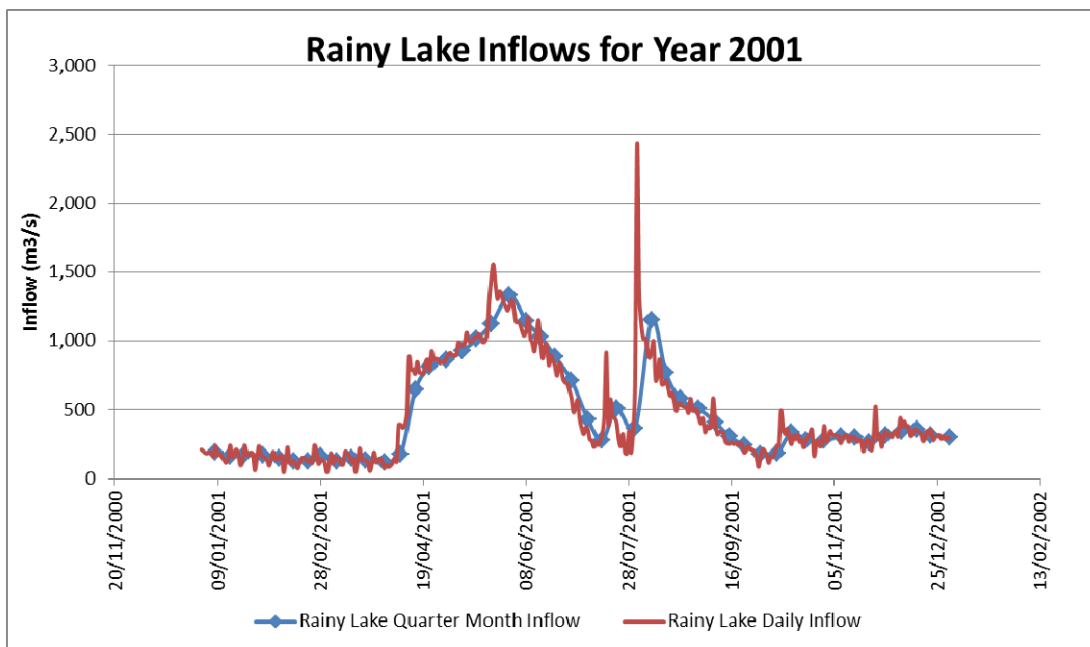


Figure 4 - Rainy Lake Inflow for 2001

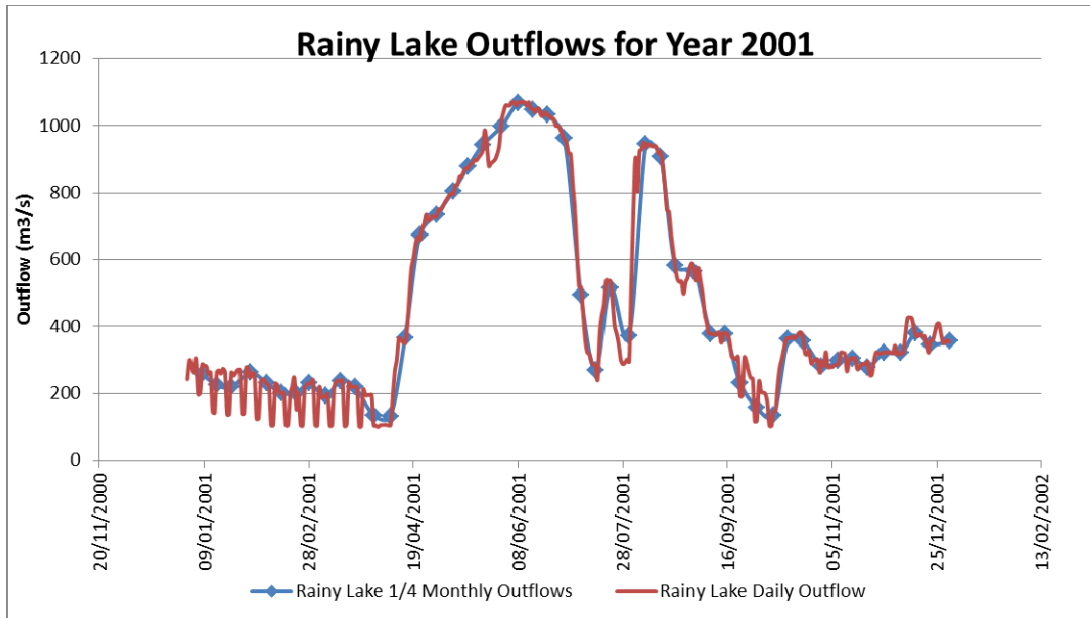


Figure 5 - Rainy Lake Outflows for Year 2001

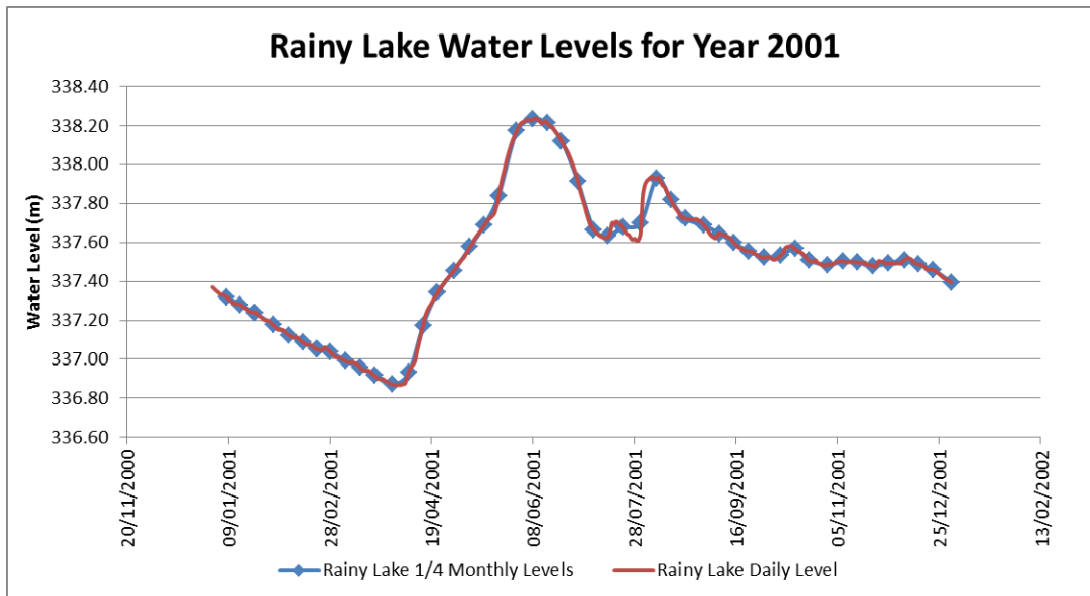


Figure 6 - Rainy Lake Water Levels for Year 2001

3.3. Model Assumptions

The purpose of the development of this model was to provide the ability to calculate the outflows and water levels in Rainy and Namakan Lakes that would have occurred historically if the 1970 and 2000 Rule Curves were followed. The development of an algorithm to calculate the outflows from the lakes that is both consistent with the regulation rules and also replicates the historic water levels of Rainy and Namakan Lakes is difficult. Outflows from Rainy and Namakan Lakes are a function of the water

levels in the lakes and the gate settings at the control structures. The gate settings are selected by the operators by taking into account the anticipated inflows to the lakes, the desired end-of-period water level and the regulation rules. Future inflows to the lakes are not known at the time the gate settings are made and therefore the end-of-period water level is also uncertain. There are human inputs into this decision making process that are not consistent over time. A numerical model of these processes must also estimate the future inflows for the interval of the calculation and follow a rigid approach for selecting the outflow that is consistent over the entire length of the simulation. The differences between the human decision making and the strict algorithms that the model uses causes differences between the observed water levels and outflows in the system and those computed by the hydrologic response model. In order to develop a consistent decision making process for the model, three simplifying assumptions were required as follows.

The first assumption is that any flow that is called for by the model that is less than the stage-discharge curve maximum can be passed by the structures using a combination of stop logs at the two dams at the outlet of Namakan Lake or the hydropower station and overflow dams at the outlet of Rainy Lake.

The second assumption is the hydrologic response model does not take into account any temporary Supplemental Orders that were issued by the Rainy Lake Board of Control. This was an assumption that was required to allow a consistent outflow decision making process to be defined and implemented in the hydrologic response model. This aspect of the model is beneficial for the purposes of the model in that it simplifies the computational algorithms the model required but it also allows the direct comparison of the 1970 and 2000 Rules Curves without the interference of short term Supplemental Orders.

The third assumption is that the 1970 Rule Curves algorithm for this model targets the middle of the Rule Curves. Without this assumption, there are infinite possibilities for the outflow from the lake. It would be possible to perhaps target the upper or lower portions of the Rule Curves to match recorded outflows but this was not the approach followed for this model. This assumption will lead to a smaller than actual difference between the 1970 and 2000 Rules Curves, especially for the period from 1970 to approximately 1986 or 1987 when the dam operators were not targeting the middle of the Rules Curves. In the late 1980's and early 1990's the dam operators started to target the middle of the Rules Curves so the computed differences between the Rules Curves would not be as conservative.

3.4. Model Inputs

While there are a number of stream flow gauges in the Rainy and Namakan Chain of Lakes system, not all of the drainage basin is gauged. The Lake of the Woods Secretariat has calculated total (gauged and un-gauged) historic inflows to both Namakan and Rainy Lakes using a similar water balance approach backwards in time. These inflows were obtained from the Lake of the Woods Secretariat on daily, weekly,

quarter-month, and half-month intervals from 1911 to 2012 for Rainy Lake and 1941 to 2012 present for Namakan Lake (M. Dewolfe, personal communication, April 6, 2012). Inflows for the years of 2013 and 2014 were also obtained from the Lake of the Woods Secretariat (J. Bomhof, personal communication, February 17, 2015). The hydrologic response model utilizes the quarter-month inflows in its computations.

The storage or volume of a lake is function of its geometry and the lake water level. This function can be described by a stage versus volume relationship or stage-volume curve. The stage-volume curve is developed by calculating the total volume of the lake at a number of different water levels and plotting the curve. The volume of the lake is calculated from bathymetric soundings and contour data either manually or using a GIS. Stage-volume curves for Rainy and Namakan Lakes were developed in about 1932 based on hydrographic surveys completed a few years earlier. These stage-volume curves are still used by the Lake of the Woods Secretariat at the current time to determine what the total storage of the lakes in relation to the measured water levels. These stage-volume curves were obtained from the Lake of the Woods Secretariat and used to relate water level and storage in Rainy and Namakan Lakes in the hydrologic response model. The tabular values are shown in Tables 1 and 2 and graphically in Figures 7 and 8. A separate component of the hydrologic response model development project investigated updating these relationships using current bathymetry and GIS technology. However, the hydrologic response model does not use the new stage-volume relationships because they were not available at the time of the development of the model. The updated stage-volume curves could be added to the model in the future. However, when the stage-volume curves in the model are replaced with new curves the inflows for the system will also have to be replaced with re-calculated inflows that also utilize the new stage-volume relationships. The hydrologic response model uses inflows that are calculated using stage-volume curves so they would also change and compensate for the difference between the existing and updated stage-volume relationships.

Using the stage-volume curves allows for the total storage to be calculated at any water surface elevation. The hydrologic response model uses linear interpolation of the tabular stage-volume relationships to compute either the storage from a specified water level or a water level from the specified storage. The change in storage for a computational time step is the difference in storage between the beginning-of-period and end-of-period.

Table 1 - Rainy Lake Stage - Volume

elevation (m)	volume (million m ³)
335.0	112.67
336.0	798.00
336.5	1176.42
337.0	1577.25
337.5	2002.06
338.0	2450.57
339.0	3416.85
340.0	4458.97

Table 2 - Namakan Lake Stage-Volume

elevation (m)	volume (million m ³)
337.0	65.33
338.0	259.95
338.5	364.20
339.0	475.58
339.5	592.46
340.0	712.28
340.5	836.56
341.0	966.17
341.5	1099.79
342.0	1239.68
343.0	1540.75

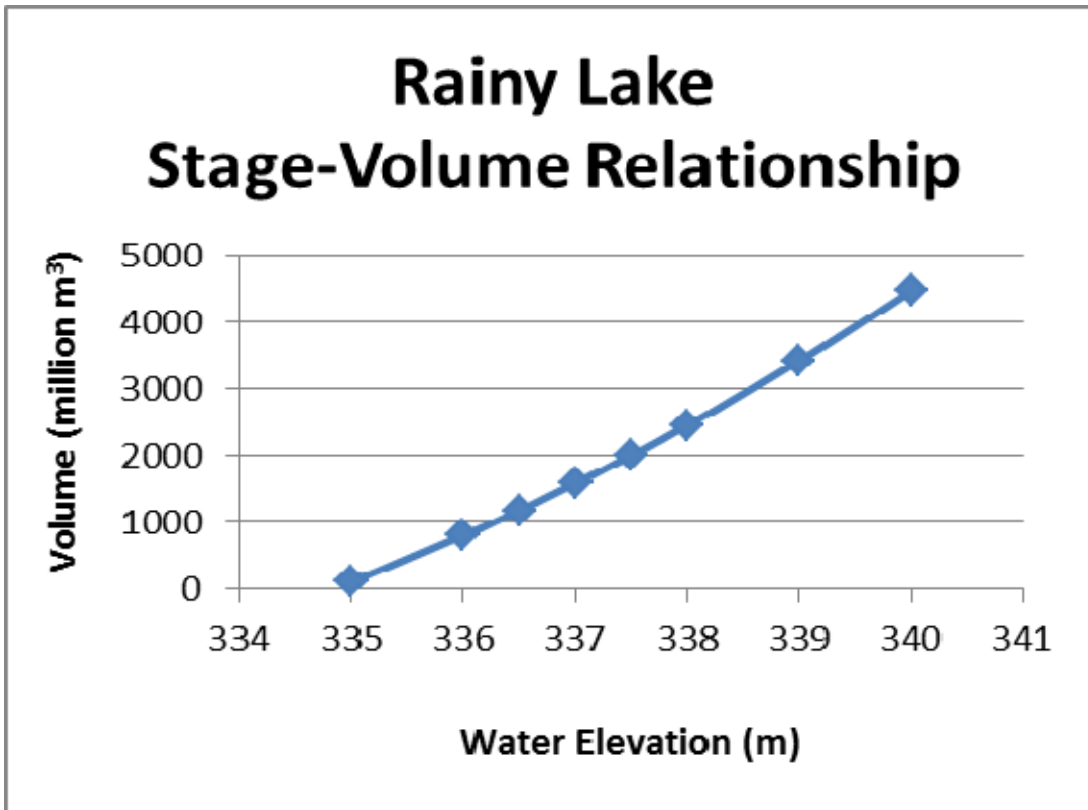


Figure 7 - Rainy Lake Stage-Volume Relationship

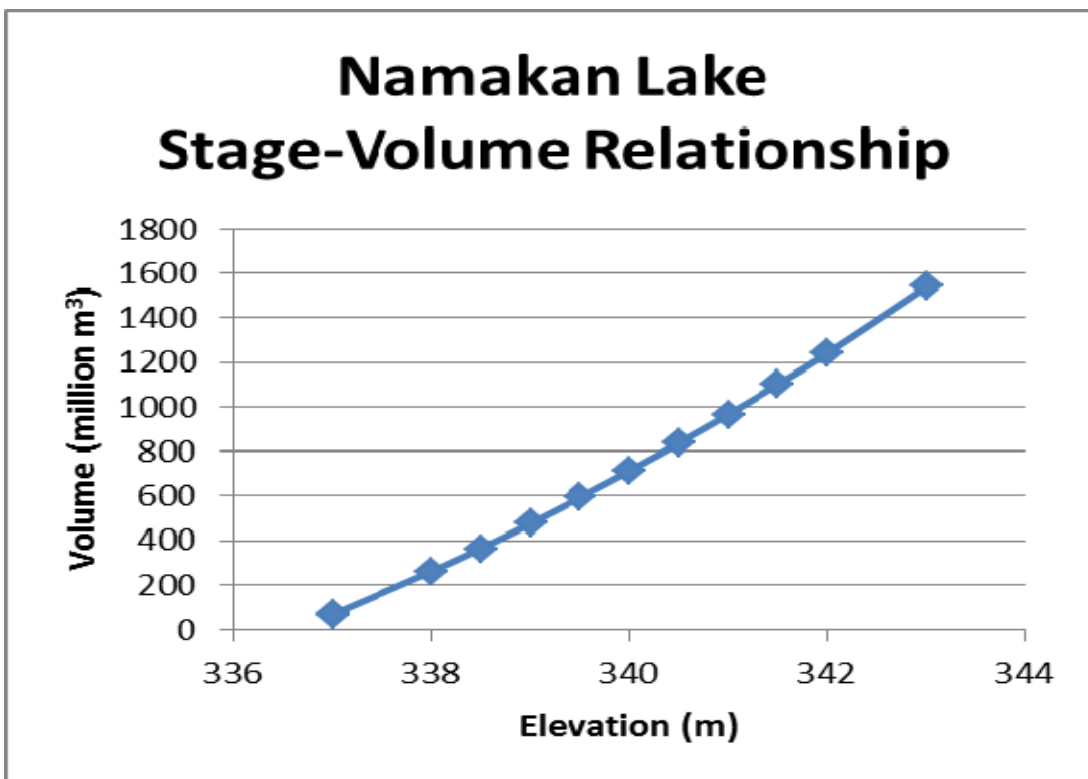


Figure 8 - Namakan Lake Stage-Volume Relationship

Stage – discharge curves for Rainy and Namakan Lakes are used to determine the maximum outflow that can be released from the Lakes for a specified water level. The dams at the outlet of Namakan Lake and at International Falls have the capacity to pass the maximum flow specified by the stage-discharge curves at all times. For Rainy Lake, the maximum outflows are determined using a single stage-discharge (M. Dewolfe, personal communication, April 6, 2012). For Namakan Lake in addition to the main channels to the Canadian and International Dams, there are two overflow channels from Namakan to Rainy Lake. Bear Portage is to east of Kettle Falls and Gold Portage is to the west. These channels permit uncontrolled flow from Namakan to Rainy Lakes when Namakan Lake is at higher lake levels. Therefore there are three stage-discharge relationships for Namakan Lake, one for the two main dams, and one for each of the two natural overflow channels. The maximum outflow from Namakan Lake for a water level is the sum of the outflows interpolated from these stage-discharge relationships.

The stage-discharge relationships were obtained from the Lake of the Woods Secretariat (M. Dewolfe, personal communication, April 6, 2012) and are shown in tabular format in Tables 3 through 6 and graphically in figures 9 through 12.

Table 3 - Rainy Lake Stage - Discharge Relationship

Elevation (m)	discharge (m ³ /s)
335.4	0
336	399
336.5	425
336.75	443
337	589
337.25	704
337.5	792
337.75	909
338	1014
338.5	1156
339	1324
339.5	1550
340	1778

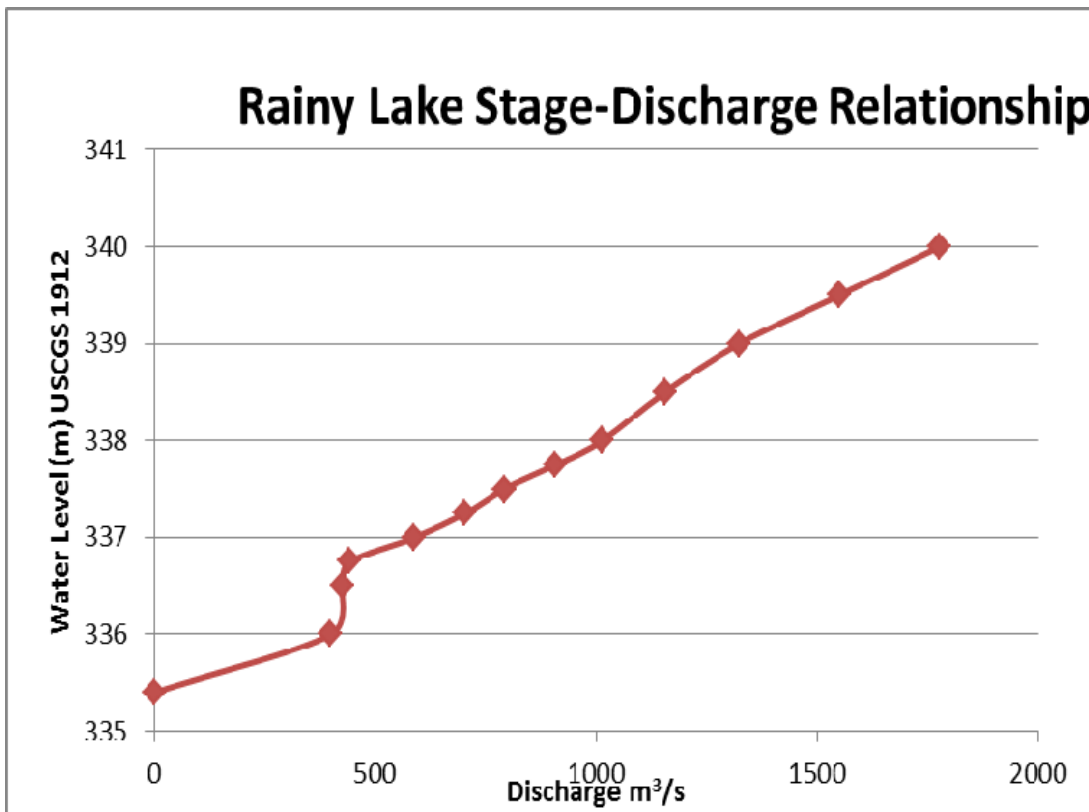


Figure 9 – Rainy Lake Stage-Discharge Relationship

Table 4 - Namakan Lake Stage-Discharge

elevation (m)	discharge (m ³ /s)
335.21	0
337	51
338	120
338.5	173
339	233
339.5	299
340	373
340.5	455
341	549
341.5	659
342	772
342.5	886
343	999

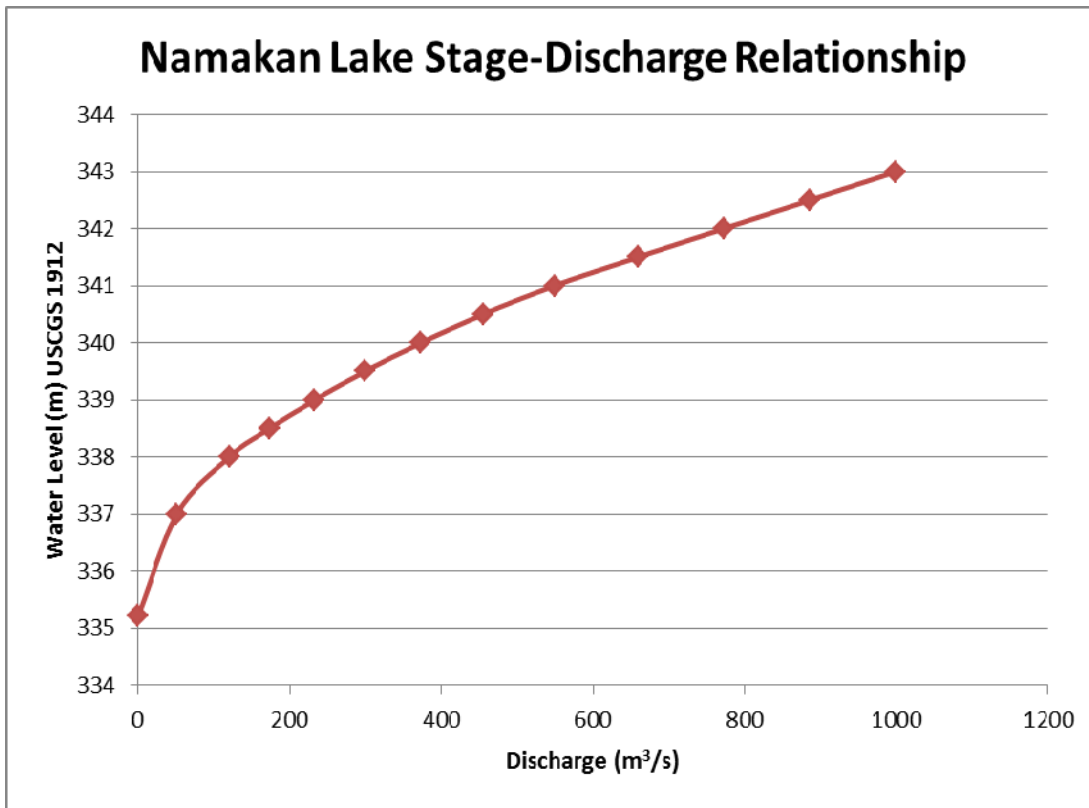


Figure 10 - Namakan Lake Stage-Discharge Relationship

Table 5 - Gold Portage Stage-Discharge

elevation (m)	discharge (m ³ /s)
336.804	0
339.285	0
339.547	0.558
339.623	0.89
340.035	4.117
340.462	10.135
340.614	12.966
340.919	19.734
341.224	27.974
342.138	55.243

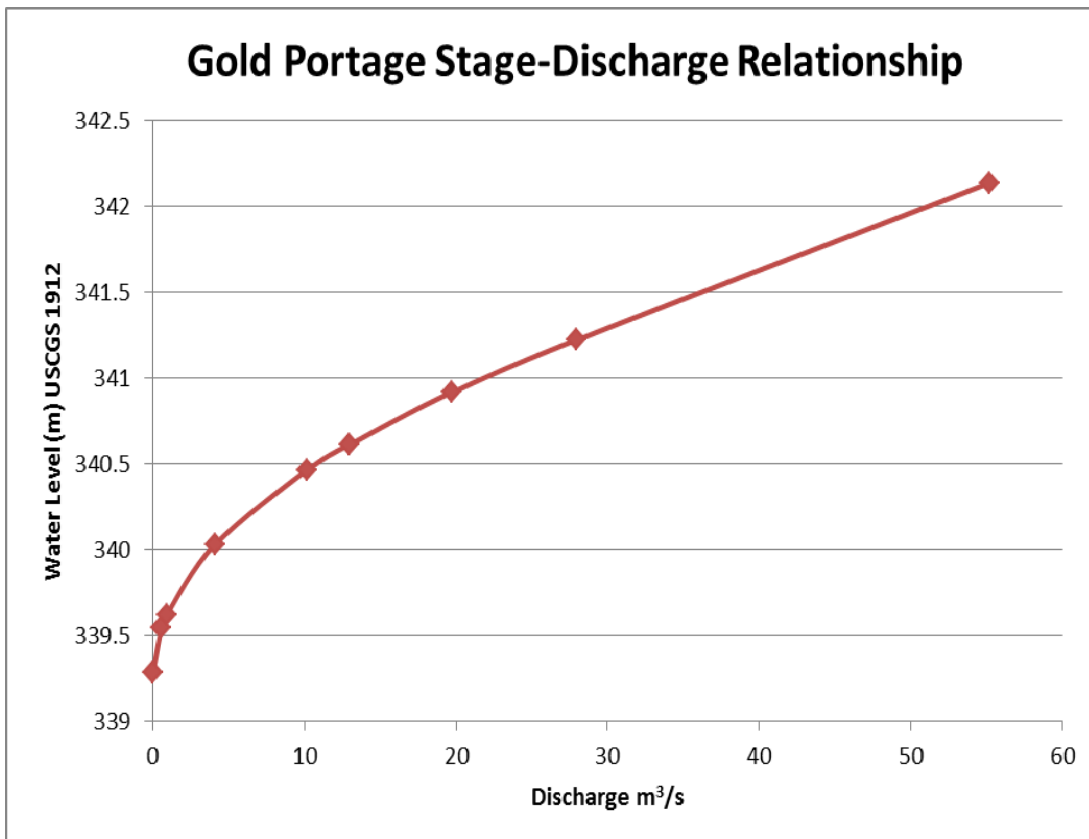


Figure 11- Gold Portage Stage-Discharge Relationship

Table 6 - Bear Portage Stage-Discharge

elevation (m)	discharge (m ³ /s)
337	0
340.39	0
340.7	0.845
340.85	2.15
340.9	2.93
341	6.23
341.1	9.7
342.5	58.327

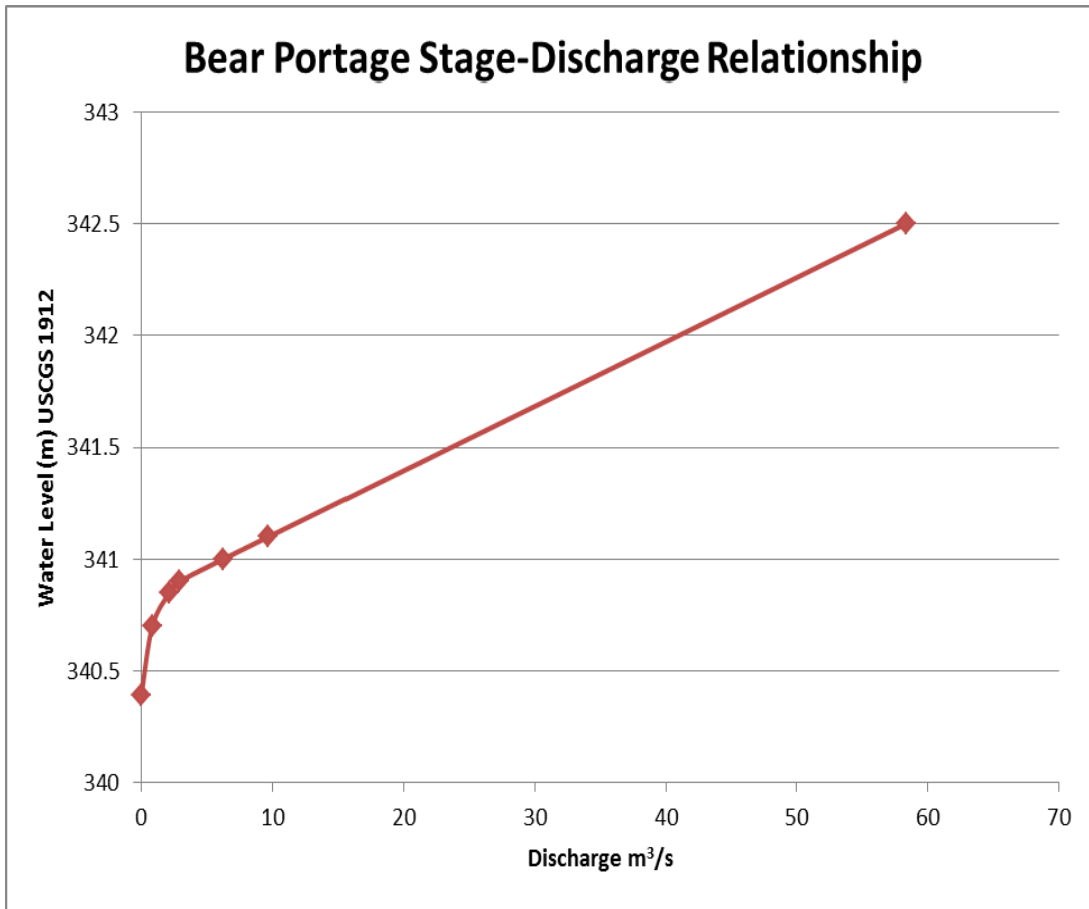


Figure 12 – Bear Portage Stage-Discharge Relationship

3.5. Model Calculations and Outflow Decision Making Methodology

The simulations of Rainy and Namakan Lake water levels and outflows are completed in four separate sub-models on four separate worksheets. There are separate worksheets for Rainy Lake 1970 Rule Curves, Rainy Lake 2000 Rule Curves, Namakan Lake 1970 Rule Curves, and Namakan Lake 2000 Rule Curves. The worksheets link to other worksheets containing the inflows and recorded levels and outflows. The Rainy worksheets link to the Namakan worksheets to obtain the computed outflow from Namakan Lake which is then used by the Rainy models for the computation of Rainy outflow for each time step.

Simulations begin with the specification of an initial water level for each lake. The worksheets then calculate the maximum outflow possible using the stage-discharge curves for each lake. The maximum outflow is determined using an iterative approach. For Namakan Lake, the total outflow is the sum of the discharges calculated from three stage-discharge relationships, one combined relationship for the two Kettle Falls dams, and one each for Bear Portage and Gold Portage. For Rainy Lake, there is only one stage-discharge relationship. The model used linear interpolation to extract the discharge for any water surface elevation from the stage-discharge tables. The beginning of period storage is also calculated using the beginning of period water level and the stage-volume relationships for the lake. The model will compute the volume from any stage by linear interpolation from the stage-volume relationship. Once the beginning of period stage is known and the initial guess of the outflow is known, the change in storage and end-of-period storage are calculated. The mean water level for the time period (beginning and end of period) is then calculated and passed to the next iteration to recalculate the outflow from the lake. This calculation is repeated five times until the outflow and end-of-period water level required to balance the equation are determined. Five iterations were sufficient for the outflow and end-of-period water level which are dependent on each other to converge at values that will not change if additional iterations were employed. This outflow is referred to as $Q_{\text{StageDischarge}}$.

Independently, the model calculates the target outflow for the period. For both the 1970 and 2000 Rule Curves models for both Rainy and Namakan Lakes the target outflow is defined as the outflow that will bring the water level to the middle of the Rule Curves level by the end of that time step. The 2000 Rule Curves for Rainy and Namakan Lake require the Power Companies to target the middle of the Rule Curves (mean of maximum Rule Curve and minimum Rule Curve water level) at all times with the exception of periods where Supplemental Orders are in place. The 1970 Rule Curves for Rainy and Namakan did not require the Power Companies to target the middle of the Rule Curves; they only had to try to stay within the Rule Curves. For this model the outflow calculation algorithm for the 1970 Rule Curves targets the middle of the Rule Curves. Without this assumption, there are mathematically infinite target water levels within the two Rule Curves so the model assumes the target is the middle of the Rule Curves. However, it would

be possible to target a lower or upper portion of the Rule Curves band to match the historic water levels if so desired, but at present the model targets the middle of the Rule Curves.

From the target water level, the model calculates the target end-of-period storage. To select an outflow, the model requires an estimate of the expected inflow for the calculation period. The inflow utilized by the hydrologic response model for Namakan Lake is the recorded quarter-month inflow as computed by the Lake of the Woods Secretariat. The inflow used in the Rainy Lake worksheets is either the recorded quarter-month inflow computed by the Lake of the Woods Secretariat, or the computed outflow from Namakan Lake for that time period. The Rainy Lake worksheets check whether the recorded inflow to Rainy Lake is larger or smaller than the outflow from Namakan Lake for the time period and if it is smaller the model utilizes the Namakan Lake outflow as the Rainy Lake inflow. Although it is possible for the inflows to Rainy Lake to be somewhat smaller than the outflows from Namakan in the late summer months and early fall when there is significantly higher evaporation from Rainy Lake, for most times in the year this is not the case (M. Dewolfe, personal communication, April 9, 2013). The outflow is then calculated using the beginning-of-period storage, inflow for the period and the target storage at the end-of-the-period. The target outflow is the outflow that results in the smallest difference of the end-of-period water level calculated with the target outflow and the recorded end-of-period water level. The end of this calculation results in the target outflow for the period.

The model then utilizes the Rule Curves and other regulation rules, the maximum outflow ($Q_{\text{STAGEDISCHARGE}}$) and the target outflow to make the decision of what the actual release will be. The model goes through a decision making process using a series of logical IF statements to decide what the outflow is. A schematic of the decisions made for each of the four models are shown in Figures 5 through 8. The model determines if the target outflow is to be released or if any of the high water level or low water level regulation rules must be applied. It does this by comparing the beginning of period water levels to the Rule Curves and determines which circumstance will apply. The Rule Curves for Rainy and Namakan Lakes were presented in section 3.1 but for illustration purposes here are a couple of examples when conditions would not allow the target outflow to be released. For example, if water levels exceed the all gates open water level on Rainy or Namakan Lakes and inflows exceed outflows, then the Power Companies must open all gates of the structures and they must remain open until these conditions subside. When this occurs, the outflow is the maximum outflow that the Lake will pass, the $Q_{\text{STAGEDISCHARGE}}$. Another example for Rainy Lake would occur when the water level falls below the 2000 Rule Curve minimum level outflows, the regulation rules state that the outflow must be set at $100 \text{ m}^3/\text{s}$. The model checks to see whether any of these Rule Curves or other regulation rules apply and if so set's the outflows according to these rules. If none of the Rule Curves or regulation rules apply, the model will release the target outflow.

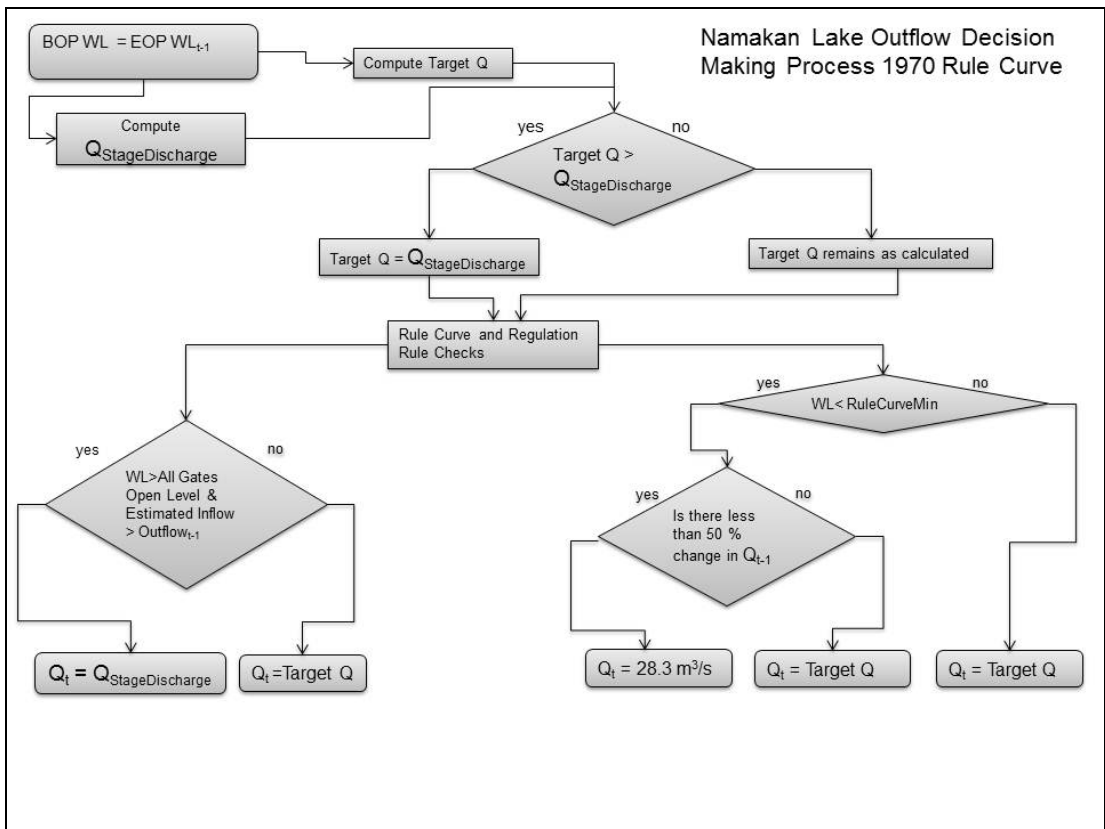


Figure 13 - Namakan Lake Outflow Decision Making Process 1970 Rule Curves

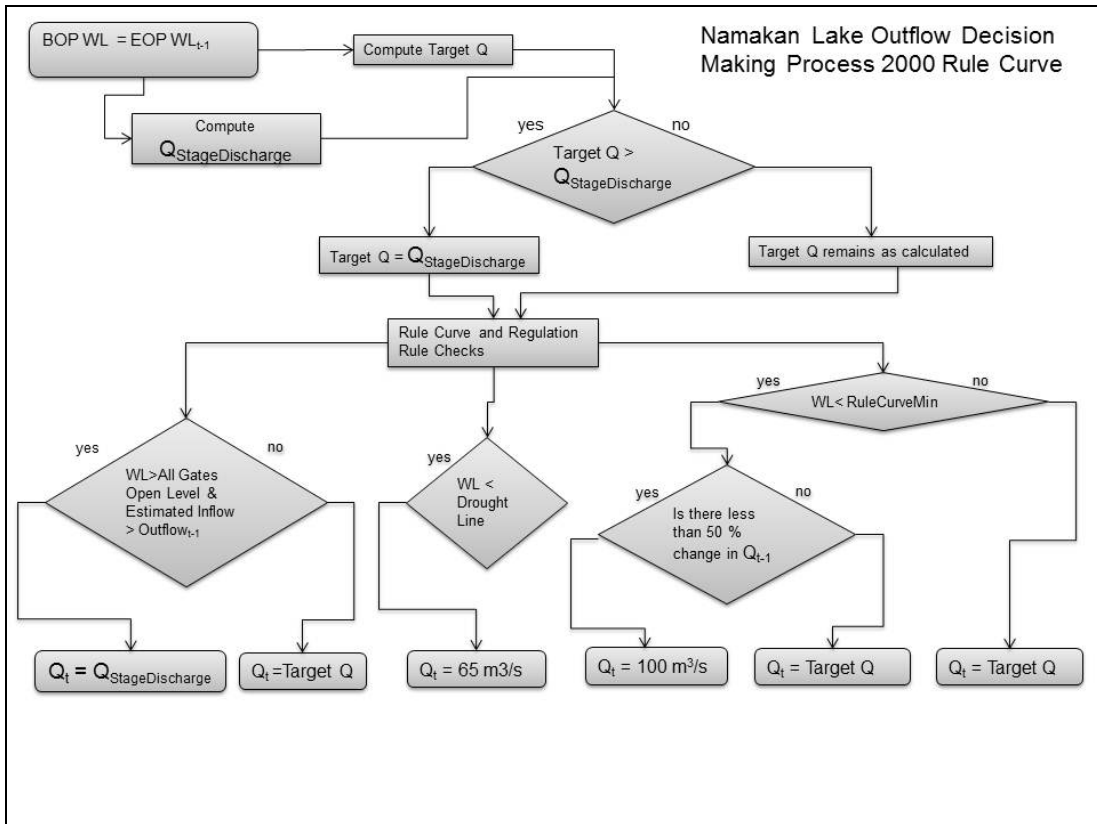


Figure 14 - Namakan Lake Outflow Decision Making Process 2000 Rule Curves

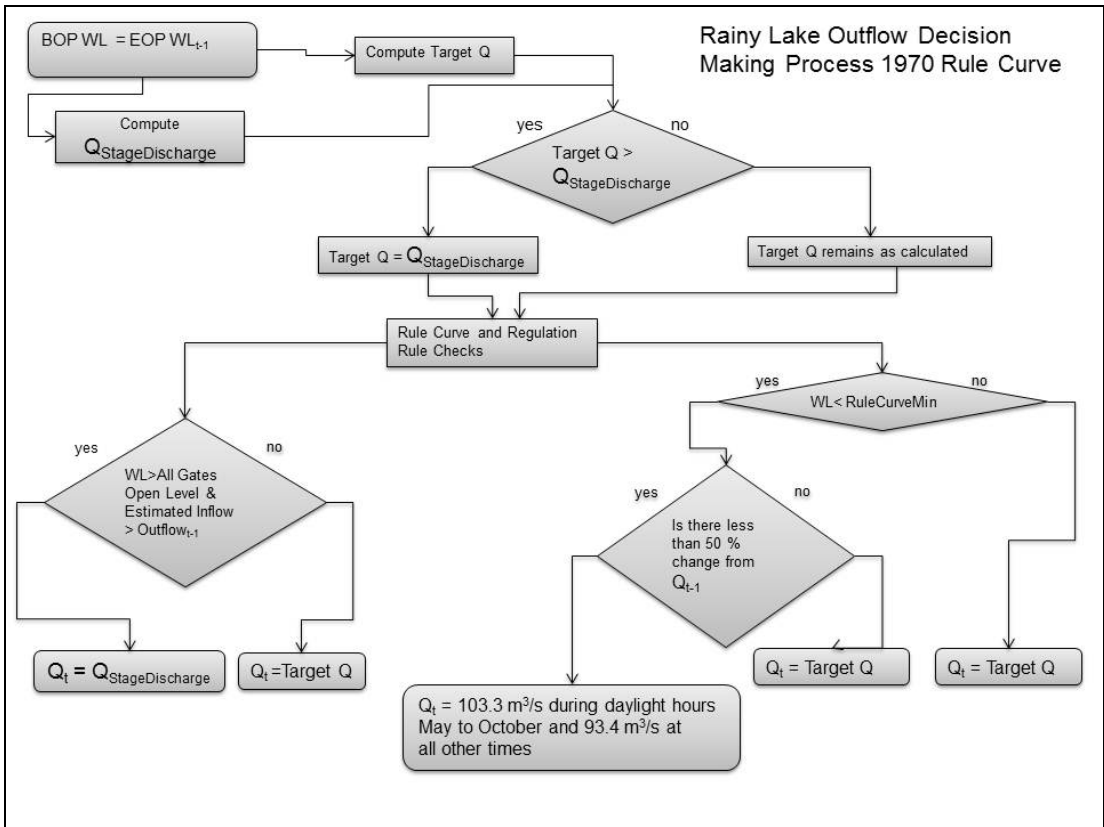


Figure 15 - Rainy Lake Outflow Decision Making Process 1970 Rule Curves

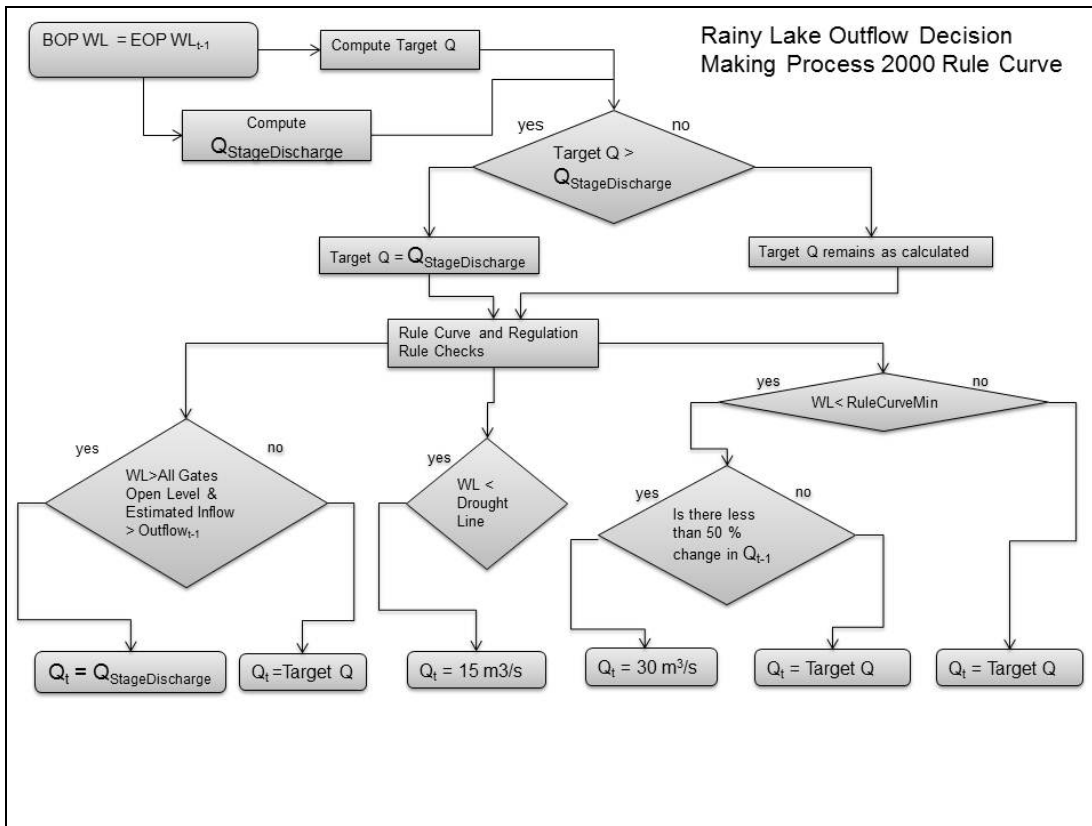


Figure 16 - Rainy Lake Outflow Decision Making Process 2000 Rule Curves

Additional computational requirements were added to the model's decision making process that do not originate from the regulation rules but were needed to get the model to work. The first of these computations was required to minimize over-reaction of the outflow decision during falling water levels and outflows. There are times when water levels at the beginning-of-period level falls below the Rule Curve minimums and as such the regulation rules specify that a minimum outflow be released. However, previous and often the following time steps call for outflows that are much bigger than the minimum. To prevent the outflows from oscillating between Rule Curve minimum and higher target flows from one time step to the next a check was added to the model that prevents the outflows from being reduced to the minimum if it will result in a more than 50 % reduction in outflow from one calculation interval to the next. Mathematically,

$$2) \text{ IF } (Q_t - Q_{min})/Q_{t-1} > 0.5 \text{ then } Q_t \text{ is the target } Q \text{ and not } Q_{min}$$

Where Q_t is the outflow for the current time step, Q_{min} is the minimum flow specified by the operating rules, and Q_{t-1} is the outflow from the previous time step. If the difference between the previous outflow and the minimum Rule Curves outflow is less than 50 %, then the outflow is set to the minimum outflow (i.e. 30 m^3/s for Namakan). While this is not included in any of the regulation rules it does reflect the reality that flows are not always reduced to the minimum flows if water

levels fall below the minimum Rule Curve level for a single interval, especially if large outflows have been required in recent time steps and inflows are large. The addition of this rule helped to smooth out the outflow calculations and allow for a smoother transition between high to average outflows and from average to minimum outflows. Figure 17 illustrates the computed outflows without this computational requirement included in the decision making of the model and Figure 18 shows the improvement in the estimation of outflows with this check. Another computation was created to handle situations in which the computed target outflow is less than the minimum outflow or a negative value. When water levels are near the higher limits of the Rule Curves and inflows are low, the target outflow calculated can be below the Rule Curve minimum outflows or even a negative value. Obviously, outflows cannot be negative or less than the Rule Curve or drought minimums, so when this occurs, the model sets the outflows to be the Rule Curve minimum.

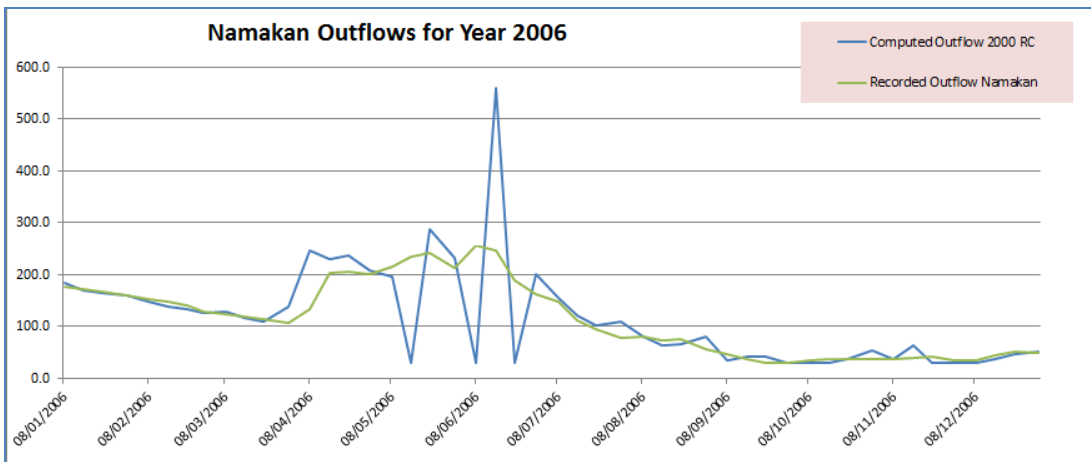


Figure 17 - Computed Namakan Outflows Without 50% Maximum Change Rule

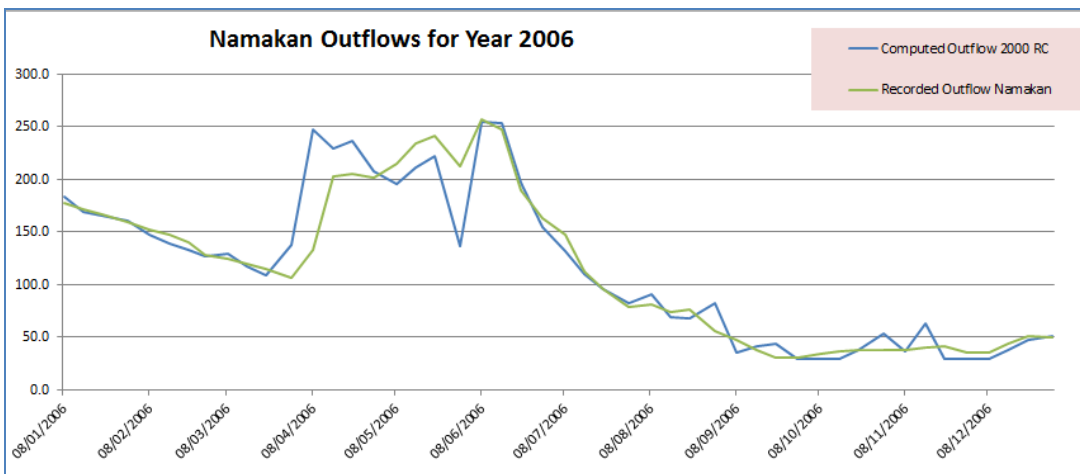


Figure 18 - Computed Namakan Outflow With 50% Maximum Change Rule

4.0. Performance of the Model

The hydrologic response model is made up of four sub-models contained on separate worksheets in the Excel spreadsheet. This section will explain how well each of the four sub-models (Namakan 1970 Rule Curves, Namakan 2000 Rule Curves, Rainy 1970 Rule Curves and Rainy 2000 Rule Curves) characterize the recorded water levels and outflows. To assess the sub-models, yearly time series graphs and residual statistics were developed and examined. The fit between computed and recorded water levels and outflows vary from year-to-year depending on the lake modelled, Rule Curves, and whether there were supplemental orders in place during the year.

4.1. Namakan 1970 Rule Curves Model

The Namakan 1970 Rule Curves model generally replicates the recorded water levels and outflows of Namakan Lake well although relative to the other three models it is the least accurate. The computed versus observed water level residuals, root mean squared errors (RMSE) and R squared are shown in Table 7. The computed versus observed outflow residuals, root mean squared errors and R squared are listed in Table 11 in the appendix. The 1970 Rule Curves for Namakan Lake were utilized from 1970 to 1999 and the mean, maximum and minimum residuals for this period were -0.01 m, 0.34 m, and -0.52 m respectively. The RMSE was 0.24 m and the R squared was 0.93. The root mean squared errors for this model were higher than the Rainy 1970 Rule Curves model because of the increased range of water levels on Namakan compared to Rainy. The root mean squared errors of the Namakan 1970 Rule Curves model are higher than the Namakan 2000 Rule Curves model because the 1970 Rule Curves did not require the hydroelectric operators to target the middle of the Rule Curves where the 2000 Rule Curves had this requirement. The 1970 Rule Curves for Namakan Lake only required the operators to remain within the Rule Curves bands. As explained earlier in this report, the model of the 1970 Rule Curves for Namakan Lake targets the middle of the rule curve as the target water level and determines the outflows based on this target level. The impact of this point is illustrated in Figure 19 and 20. Figure 19 shows the computed and recorded water levels on Namakan Lake for 1974 and Figure 20 shows the computed and recorded outflows. At the beginning of the year, the modelled water levels are near the middle of the rule curve band while the recorded water levels are just above the minimum of the Rule Curves band. The hydroelectric operators were operating the outlet of Namakan Lake according to the Rule Curves but obviously not targeting the middle of the Rule Curves. By spring and for the remainder of that year, the computed and recorded water levels match well. There were no supplemental orders in effect in 1974.

Table 7 - Namakan Lake 1970 Rule Curves Model Water Level Statistics

Year	Mean Residual (m)	Maximum Residual (m)	Minimum Residual (m)	Root Mean Squared Error (m)	R ²
1970	0.00	0.27	-0.46	0.20	0.96
1971	0.12	0.58	-0.35	0.28	0.97
1972	0.17	0.56	-0.24	0.27	0.98
1973	-0.07	0.32	-0.51	0.19	0.95
1974	0.13	0.51	-0.20	0.26	0.98
1975	0.07	0.29	-0.24	0.13	0.99
1976	0.08	0.59	-0.29	0.22	0.92
1977	0.04	0.65	-0.33	0.29	0.94
1978	0.08	0.49	-0.67	0.30	0.93
1979	0.00	0.27	-0.24	0.11	0.98
1980	-0.27	0.29	-0.89	0.44	0.70
1981	-0.04	0.30	-0.89	0.31	0.85
1982	0.01	0.25	-0.68	0.21	0.94
1983	-0.04	0.17	-0.42	0.15	0.97
1984	0.00	0.34	-0.64	0.23	0.92
1985	-0.06	0.19	-0.56	0.22	0.94
1986	0.10	0.44	-0.64	0.28	0.90
1987	0.15	0.42	-0.06	0.20	0.97
1988	0.02	0.40	-0.66	0.26	0.91
1989	-0.01	0.27	-0.79	0.26	0.90
1990	-0.12	0.27	-0.84	0.32	0.89
1991	-0.12	0.21	-0.67	0.28	0.93
1992	-0.10	0.21	-0.71	0.26	0.93
1993	-0.07	0.29	-0.59	0.25	0.93
1994	-0.05	0.24	-0.72	0.24	0.92
1995	-0.01	0.20	-0.31	0.14	0.98
1996	-0.07	0.23	-0.53	0.18	0.97
1997	-0.05	0.24	-0.51	0.21	0.95
1998	-0.03	0.31	-0.46	0.19	0.91
1999	-0.07	0.26	-0.54	0.22	0.92
Mean of yearly residuals	-0.01	0.34	-0.52	0.24	0.93

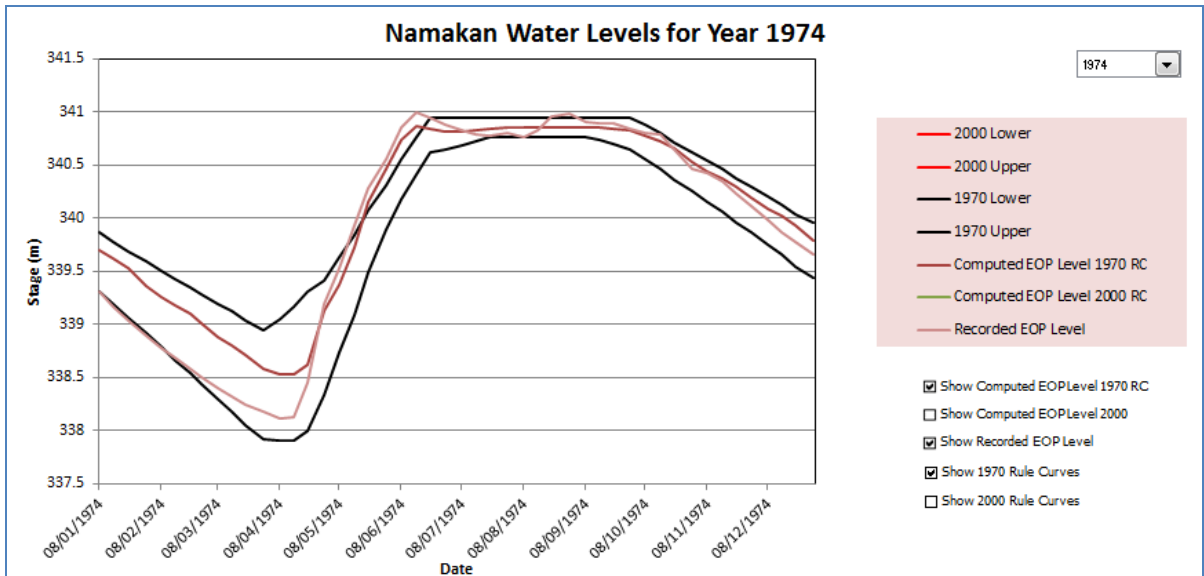


Figure 19 - Computed Versus Recorded Water Levels for 1974

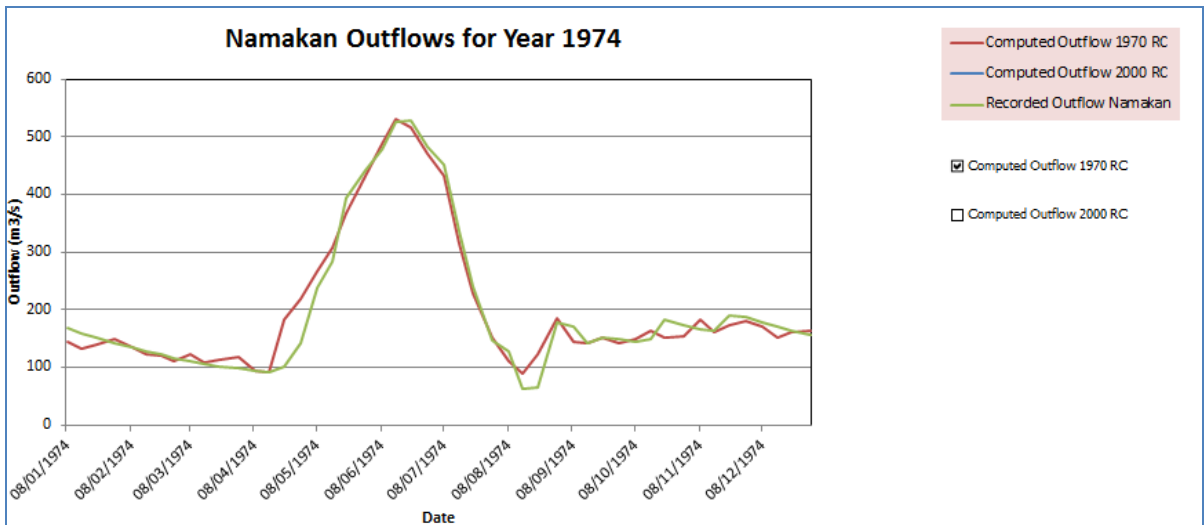


Figure 20 - Computed Versus Recorded Outflows for 1974

4.2. Namakan 2000 Rule Curves Model

The Namakan 2000 Rule Curves model replicates the recorded water levels and outflows well and better than the Namakan 1970 Rule Curves model primarily because both the Rule Curves and the model target the middle of the Rule Curves bands. The computed versus observed water level residuals, root mean squared errors and R squared are shown in Table 8. The 2000 Rule Curves for Namakan Lake were implemented starting on January 18th, 2000 and were computed in the hydrologic response model to the end of 2012. The mean, maximum and minimum residuals for this period were -0.01 m, 0.14 m, and -0.38 m respectively. The RMSE was 0.13 m and the R squared was 0.93. The computed versus

observed outflow residuals, root mean squared errors and R squared are listed in Table 12 in the appendix.

Table 8 - Namakan Lake 2000 Rule Curves Model Water Level Goodness of Fit

Year	Mean Residual (m)	Maximum Residual (m)	Minimum Residual (m)	Root Mean Squared Error (m)	R ²
2000	-0.05	0.09	-0.44	0.16	0.88
2001	-0.04	0.11	-0.47	0.13	0.95
2002	-0.04	0.08	-0.41	0.14	0.94
2003	0.05	0.16	-0.05	0.07	0.99
2004	-0.03	0.17	-0.33	0.14	0.92
2005	-0.08	0.07	-0.54	0.19	0.86
2006	0.00	0.17	-0.53	0.20	0.82
2007	0.07	0.30	-0.15	0.12	0.97
2008	-0.02	0.09	-0.48	0.13	0.94
2009	0.01	0.10	-0.25	0.08	0.98
2010	0.01	0.23	-0.21	0.10	0.96
2011	-0.01	0.11	-0.41	0.14	0.90
2012	-0.06	0.10	-0.53	0.17	0.88
2013	0.01	0.27	-0.45	0.11	0.96
2014	0.01	0.26	-0.34	0.11	0.97
Mean of yearly residuals	-0.01	0.14	-0.38	0.13	0.93

Figures 21 and 22 illustrate one year of computed versus recorded water levels and computed versus recorded outflows. In the spring and summer of 2001 there were high supplies which resulted in high levels on Namakan Lake. The operators of the structures at the outlet of Namakan Lake likely did not expect the supplies that were received in the spring of 2001, where the hydrologic response model utilizes the recorded inflows to calculate the outflows. Therefore, the model increases outflows earlier than the operators actually did that year. However, the maximum levels in the summer computed and recorded were similar.

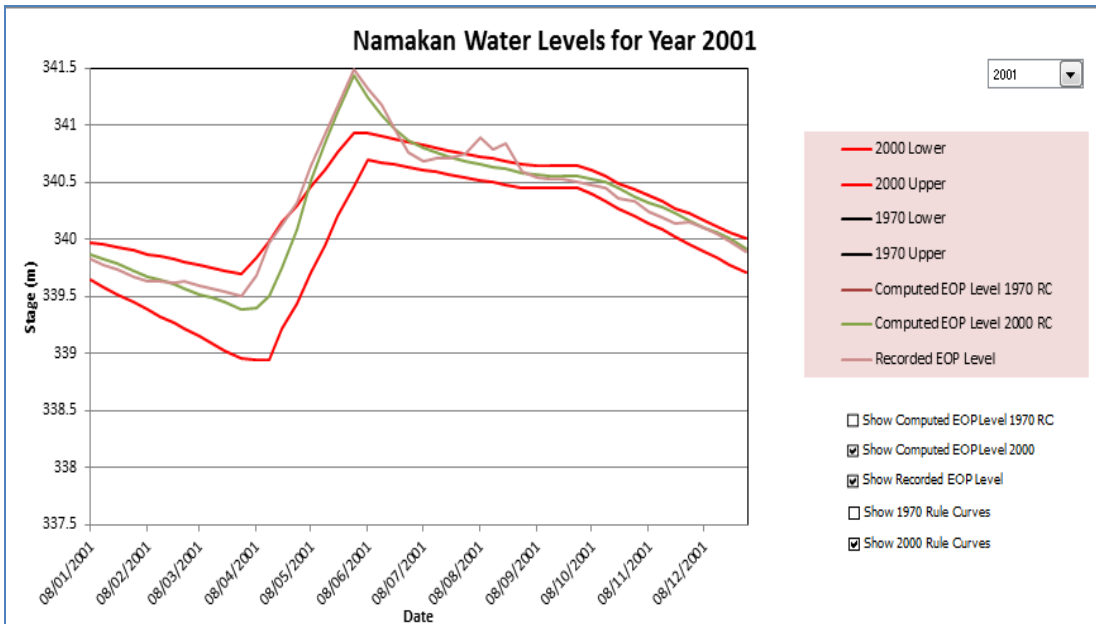


Figure 21 - Computed Versus Recorded Water Levels for 2001

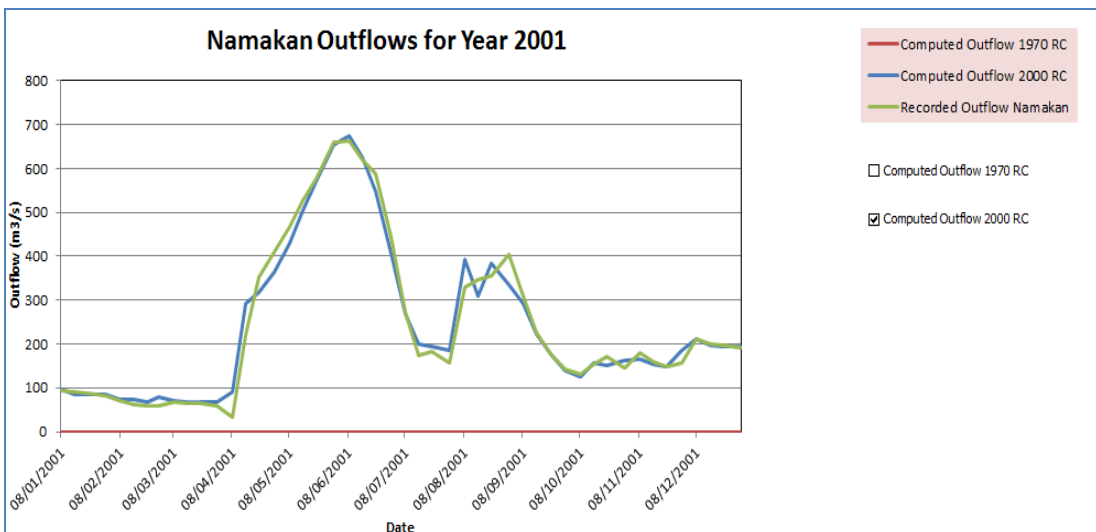


Figure 22 - Computed Versus Recorded Outflows for 2001

4.3 Rainy 1970 Rule Curves Model

The Rainy 1970 Rule Curves model does not replicate the recorded water levels and outflows as well as the Namakan 1970 Rule Curves in terms of R squared but the RMSE of the model was smaller. The mean, maximum and minimum residuals for this period were 0.01 m, 0.19 m, and -0.27 m respectively. The root mean squared error was 0.13 m and the R squared was 0.85. The RMSE for the Rainy 1970 Rule Curves model is smaller due to the smaller range of water levels that occur on Rainy Lake than Namakan Lake. The computed versus observed water level residuals, root mean squared errors and R squared for this model are shown in Table 9. For most years the R squared is greater than 0.9 but for some

years the R squared is quite low, for instance in 1980 and 1991. Figures 23 and 24 illustrate water levels and outflows for Rainy Lake in 1980. That year there were extremely dry conditions in the basin and water levels were low. A supplemental order was issued to reduce the minimum outflows from Rainy Lake that year below those specified in the Rule Curves. The hydrologic response model continues to release the minimum outflows as specified by the Rule Curves operating rules and therefore produces lower water levels than were recorded in the summer and fall. In 1991, the operators maintained water levels on Rainy Lake within the Rule Curves limits in the winter and spring but were operating at the lower and upper extremes of the Rule Curves. The hydrologic response model targeted the middle of Rule Curves during this period so the match between computed and observed was not good during this period. Later in the year the recorded water levels were lower than the computed. The computed versus observed outflow residuals, root mean squared errors and R squared are listed in Table 13 in the appendix.

Table 9 - Rainy Lake 1970 Rule Curves Model Residuals

Year	Mean Residual (m)	Max Residual (m)	Min Residual (m)	Root Mean Squared Error (m)	R ²
1970	0.05	0.42	-0.22	0.16	0.90
1971	0.00	0.11	-0.27	0.08	0.93
1972	0.07	0.39	-0.18	0.14	0.93
1973	0.01	0.13	-0.08	0.06	0.96
1974	0.04	0.21	-0.20	0.10	0.94
1975	0.07	0.23	-0.14	0.11	0.91
1976	0.05	0.19	-0.30	0.15	0.74
1977	0.04	0.20	-0.21	0.13	0.90
1978	0.05	0.22	-0.15	0.11	0.93
1979	0.08	0.20	-0.11	0.12	0.92
1980	-0.11	0.19	-0.49	0.24	0.55
1981	-0.01	0.13	-0.35	0.14	0.71
1982	0.00	0.22	-0.38	0.14	0.85
1983	-0.01	0.15	-0.16	0.08	0.95
1984	0.03	0.16	-0.23	0.11	0.88
1985	0.01	0.17	-0.14	0.08	0.95
1986	0.02	0.17	-0.40	0.13	0.80
1987	0.04	0.19	-0.32	0.15	0.83
1988	0.00	0.31	-0.20	0.12	0.87
1989	-0.05	0.14	-0.43	0.14	0.83
1990	-0.03	0.16	-0.38	0.13	0.75
1991	0.01	0.12	-0.31	0.14	0.67
1992	-0.03	0.10	-0.32	0.11	0.84
1993	0.02	0.17	-0.25	0.11	0.86
1994	0.04	0.21	-0.23	0.13	0.90
1995	0.02	0.19	-0.12	0.09	0.93
1996	0.02	0.26	-0.18	0.10	0.94
1997	0.03	0.14	-0.22	0.10	0.81
1998	-0.12	0.13	-0.49	0.21	0.83
1999	-0.10	0.15	-0.51	0.19	0.82
Mean of Yearly Residuals	-0.01	0.19	-0.27	0.13	0.85

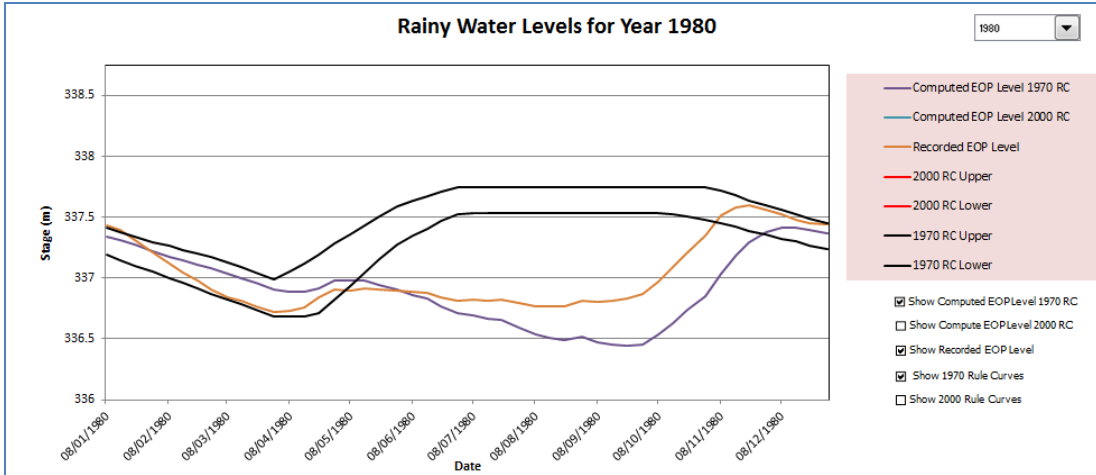


Figure 23 - Computed Versus Recorded Water Levels for 1980

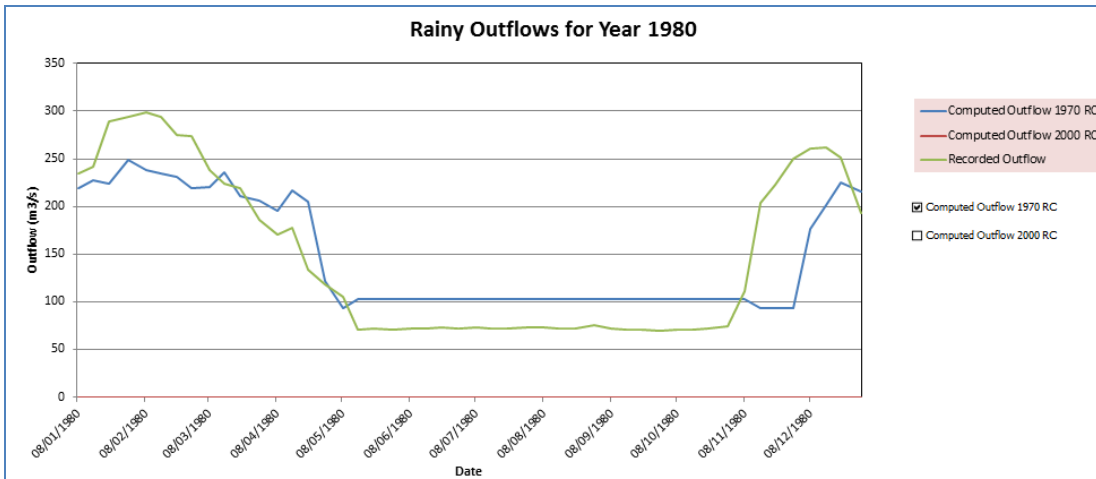


Figure 24 - Computed Versus Recorded Outflows for 1980

4.4. Rainy 2000 Rule Curves Model

The Rainy 2000 Rule Curves model similar to the Namakan 2000 Rule Curves model has a lower RMSE and higher R squared than the Rainy 1970 Rule Curves model. This is again due to the closer match between target water levels used in the model and specified in the Rule Curves operating rules. The computed versus observed water level residuals, root mean squared errors and R squared are shown in Table 10. The mean, maximum and minimum residuals for this period were -0.01 m, 0.10 m, and -0.22 m respectively. The RMSE was 0.09 m and the R squared was 0.92. Figures 25 and 26 show the computed versus observed water levels and outflows for Rainy Lake for 2002. The computed versus observed outflow residuals, root mean squared errors and R squared are listed in Table 14 in the appendix.

Table 10 - Rainy Lake 2000 Rule Curves Model Residuals

Year	Mean Residual (m)	Max Residual (m)	Min Residual (m)	Root Mean Squared Error (m)	R ²
2000	-0.04	0.05	-0.23	0.09	0.89
2001	-0.03	0.07	-0.22	0.08	0.96
2002	0.01	0.12	-0.20	0.08	0.97
2003	0.08	0.17	0.02	0.09	0.97
2004	-0.03	0.11	-0.27	0.10	0.86
2005	-0.03	0.08	-0.28	0.10	0.88
2006	0.01	0.16	-0.26	0.11	0.81
2007	-0.07	0.07	-0.34	0.13	0.91
2008	-0.02	0.08	-0.32	0.10	0.91
2009	-0.02	0.08	-0.14	0.07	0.94
2010	0.01	0.09	-0.16	0.05	0.97
2011	0.01	0.11	-0.25	0.08	0.88
2012	-0.05	0.05	-0.26	0.10	0.90
2013	0.00	0.13	-0.27	0.08	0.96
2014	-0.01	0.14	-0.16	0.07	0.99
Mean of Yearly Residuals	-0.01	0.10	-0.22	0.09	0.92

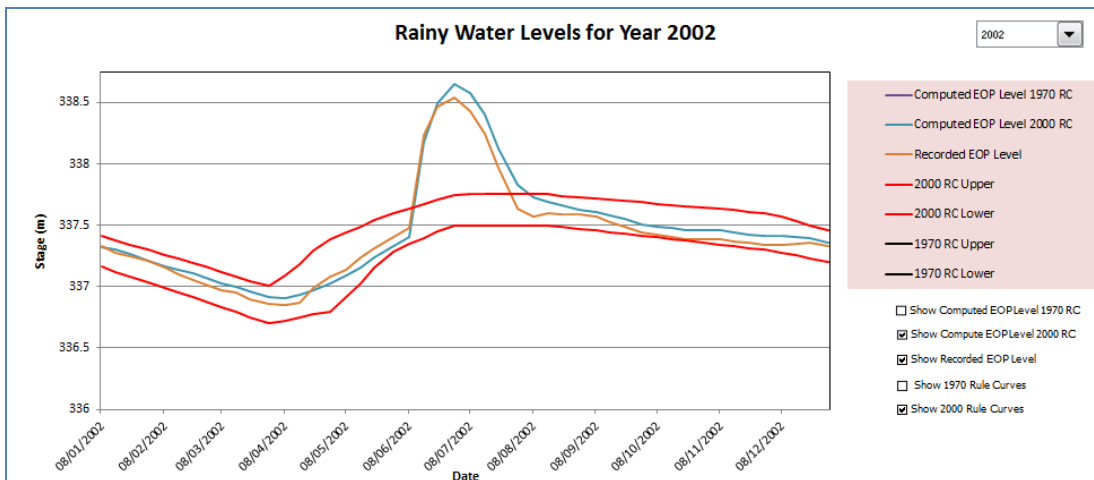


Figure 25- Computed Versus Recorded Water Levels for 2002

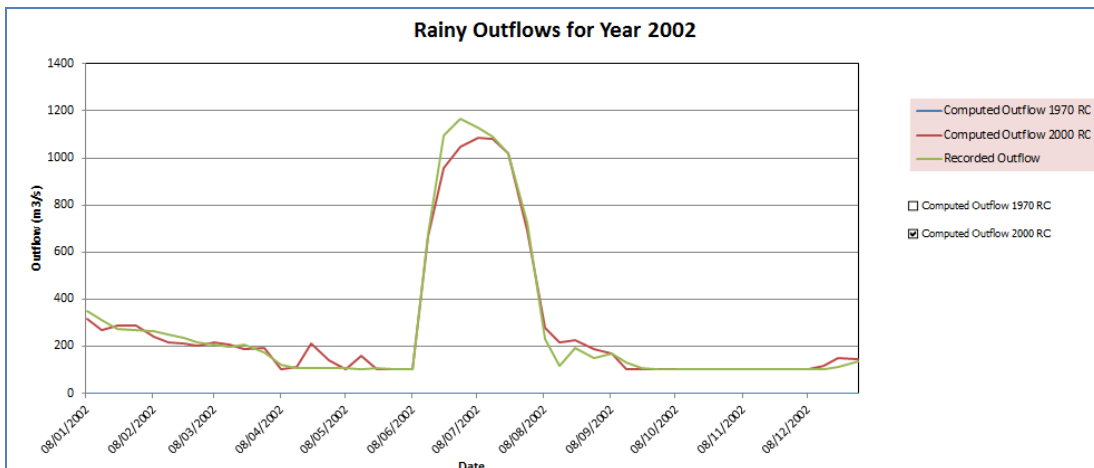


Figure 26- Computed Versus Recorded Outflows for 2002

5.0. Time Series and Stage-Frequency Curves

This section contains figures of the computed time series, monthly histograms and stage-frequency curves for the two Rule Curves regulation strategies. Additional figures and tabular data can be obtained from the Excel model.

5.1 Time Series

The computed time series of water levels from 1950 to 2012 for Rainy and Namakan Lakes are shown in Figures 27 through 32. The tabular time series may be obtained from the Excel model.

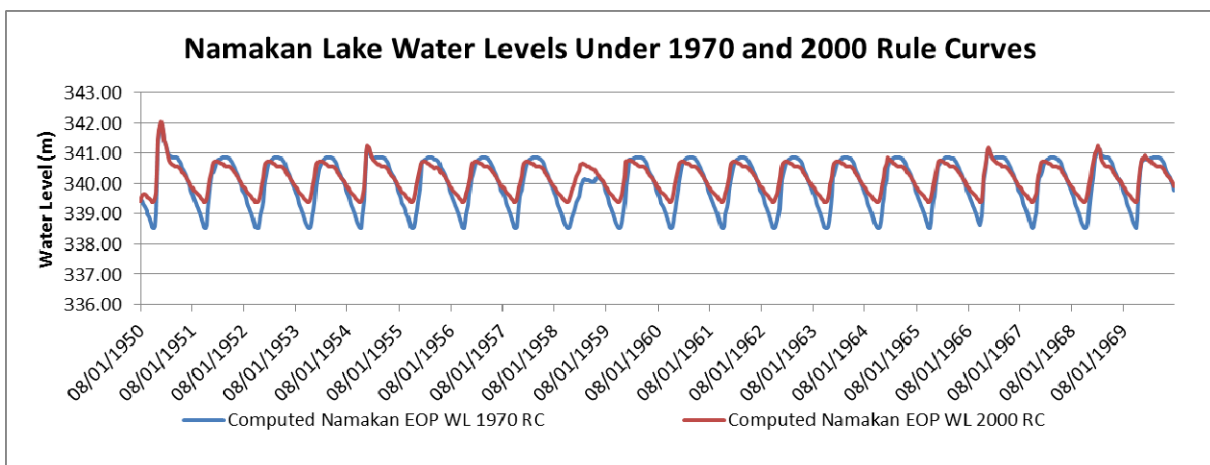


Figure 27– Namakan Lake Time Series Computed Water Level 1950-1969

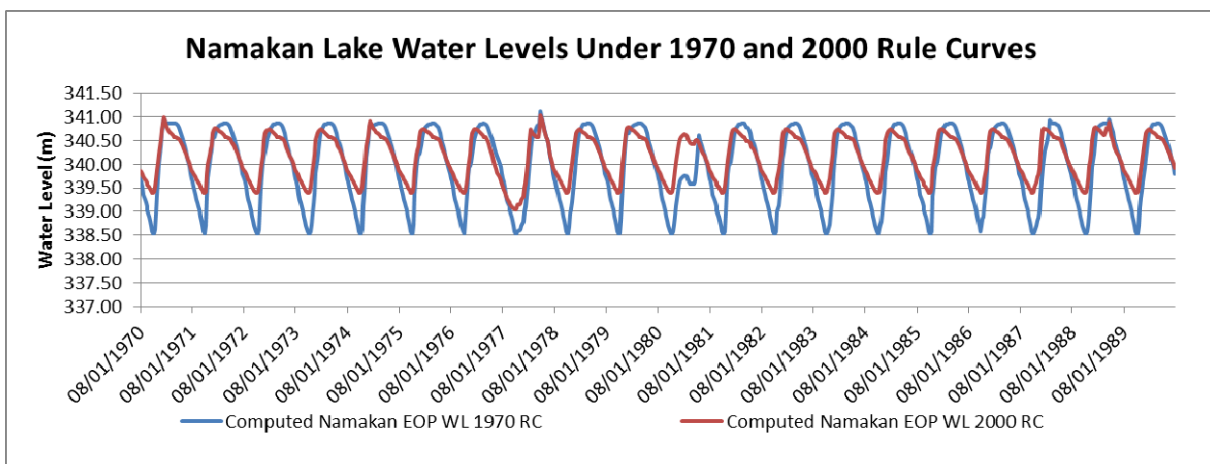


Figure 28 – Namakan Lake Time Series Computed Water Level 1970 – 1989

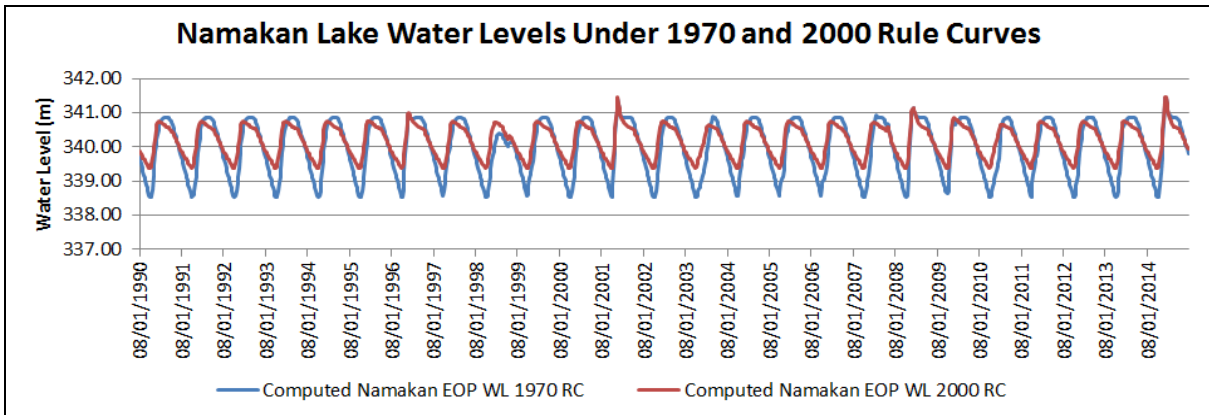


Figure 29 - Namakan Lake Time Series Computed Water Level 1990-2014

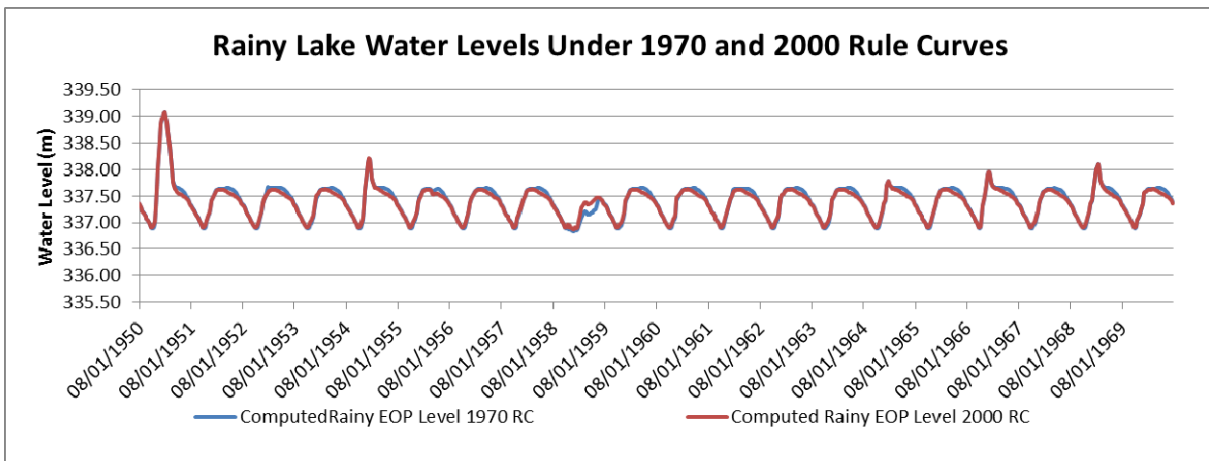


Figure 30 - Rainy Lake Time Series Computed Water Level 1950-1969

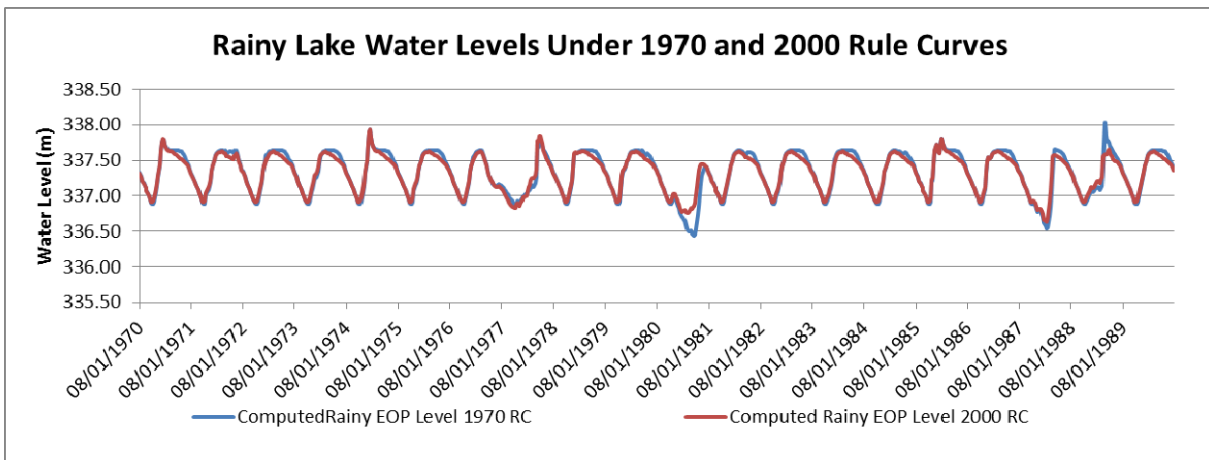


Figure 31 - Rainy Lake Time Series Computed Water Level 1970-1989

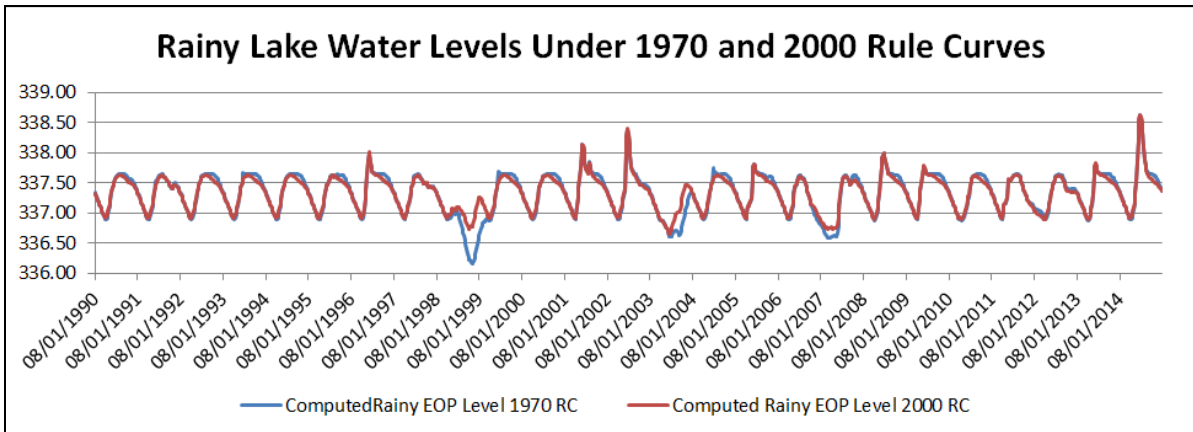


Figure 32 - Rainy Lake Time Series Computed Water Level 1990-2014

5.1 Stage-Frequency Curves

The model will also plot histograms and stage versus frequency curves for each lake for each month of the year. These may be of use to the subsequent investigators of the Rainy and Namakan 2015 Rule Curves review. The histograms plot the number times water levels were within defined ranges for each month over the 1950 to 2014 time period. The model will generate a histogram for each month using the calculated water level time series. The histograms illustrate which months have the largest difference in water levels under the 1970 and 2000 Rule Curves regulation strategies. Two examples of the histograms are shown in Figures 33 and 34.

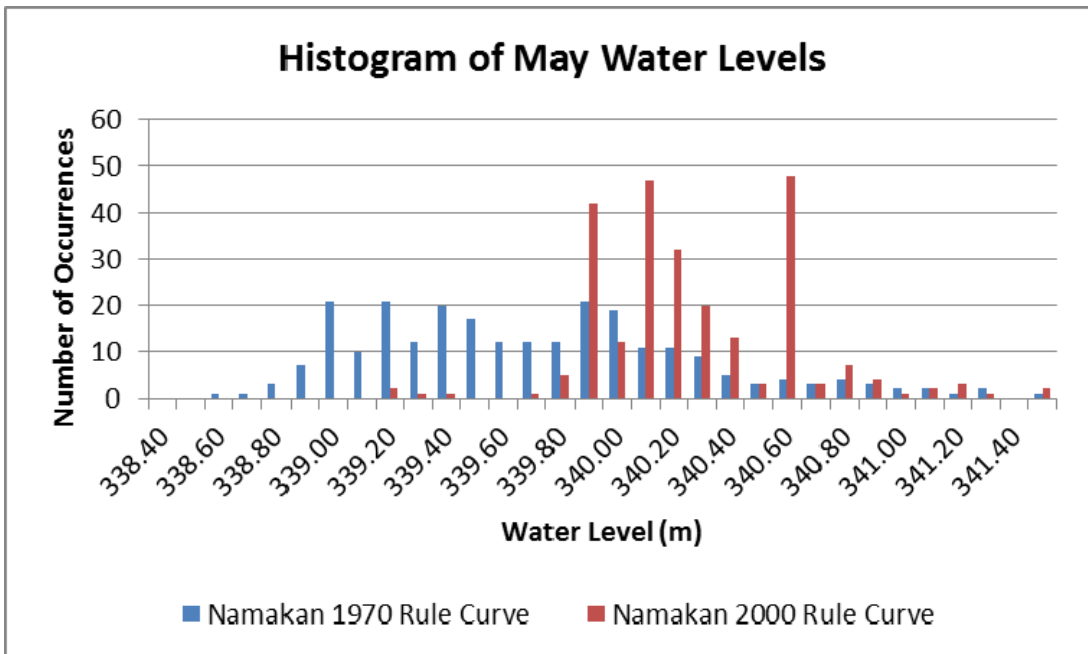


Figure 33 - Histogram of May Water Levels for Namakan Lake

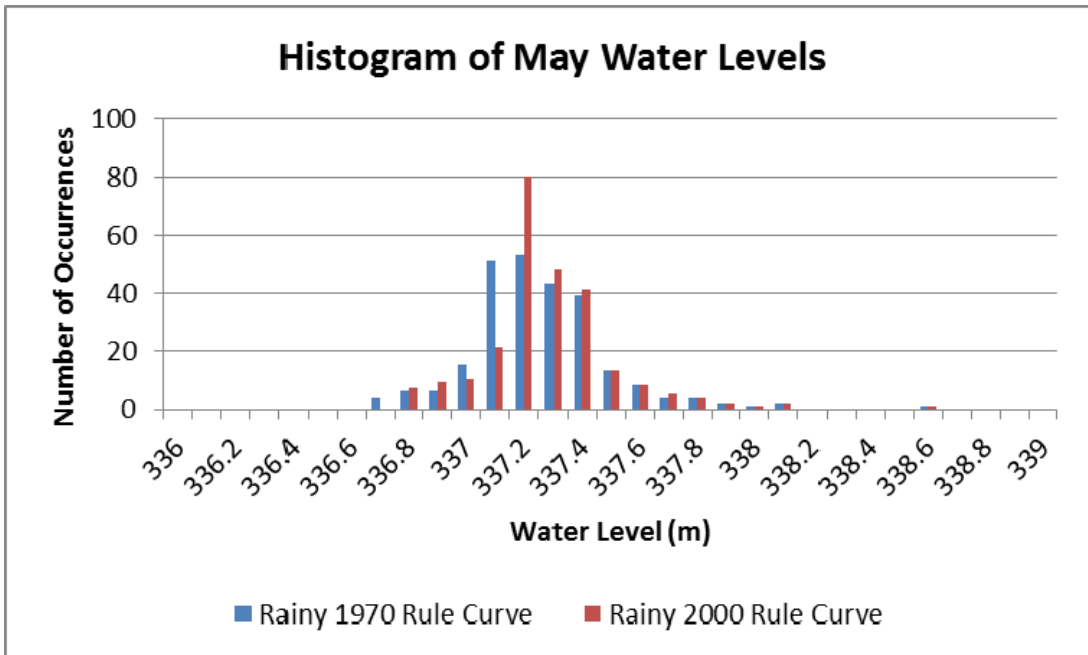


Figure 34 - Histogram of May Water Levels for Rainy Lake

A stage-frequency curve plots the percentage of time water levels exceed a given stage. The percentage of exceedance is plotted on the vertical axis and the water levels are plotted on the horizontal axis. Stage-frequency curves were generated by month from the computed water level time series under both rating curves. The model will generate a stage-frequency plot for each month. Two examples of the stage-frequency plots are shown in Figures 35 and 36.

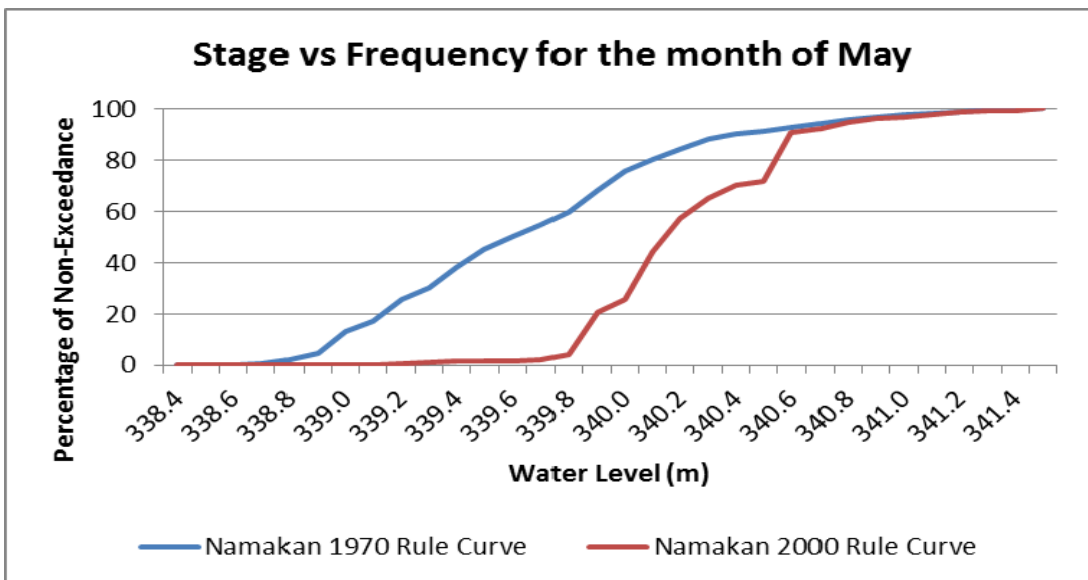


Figure 35 - Stage versus Frequency for the Month of May for Namakan Lake

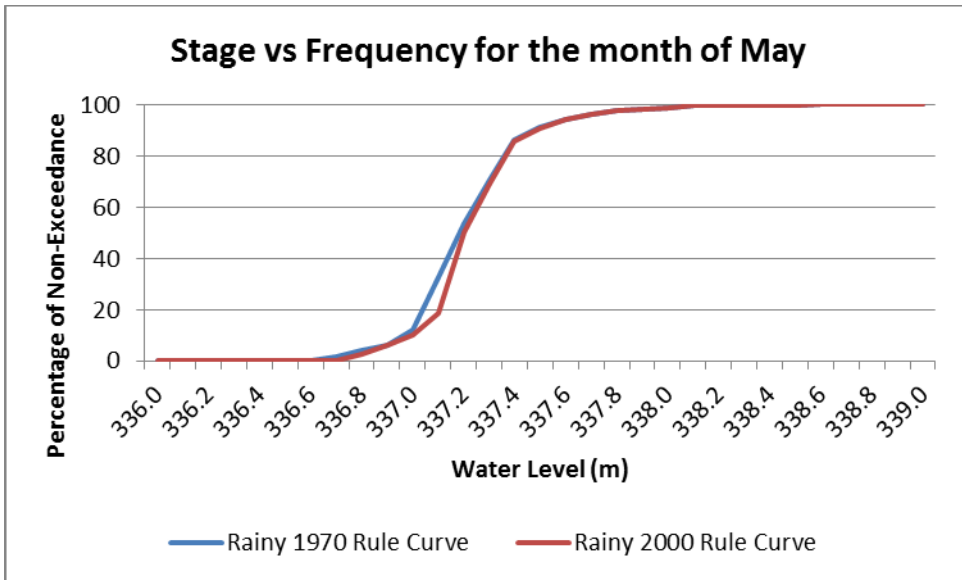


Figure 36 - Stage versus Frequency for the Month of May Rainy Lake

References

Kallemeyn, L., W.R. Darby, E. Eaton, K.Peterson, K. Smokorowski, and J. Van den Broeck. 2009. Plan of study for the evaluation of the IJC 2000 Order for Rainy and Namakan lakes and Rainy River. Unpublished report prepared for the International Joint Commission by the 2000 Rule Curves Assessment Workgroup, 55+ pp.

Linsley, R.K., M.A. Kohler, and J.L.H. Paulhus, 1982, Hydrology for Engineers, McGraw Hill, 508 pp.

M. Dewolfe, Lake of the Woods Secretariat, Personal Communication, April 6, 2012

M. Dewolfe, Lake of the Woods Secretariat, Personal Communication, April 9, 2013)

Appendix

Table 11- Namakan Lake 1970 Rule Curves Model Outflow Statistics

Year	Mean Residual (m3/s)	Max Residual (m3/s)	Min Residual (m3/s)	Root Mean Squared Error (m3/s)	R ²
1970	-2	96	-64	33	0.96
1971	1	118	-53	29	0.95
1972	0	50	-58	24	0.89
1973	-2	41	-62	22	0.87
1974	1	80	-32	23	0.97
1975	-1	48	-67	20	0.95
1976	-2	107	-120	33	0.87
1977	4	45	-40	17	0.99
1978	0	112	-87	32	0.90
1979	1	108	-44	23	0.97
1980	-2	85	-103	32	0.64
1981	0	85	-70	31	0.92
1982	0	118	-80	34	0.90
1983	1	41	-114	24	0.73
1984	1	71	-86	27	0.89
1985	-2	66	-65	26	0.95
1986	0	74	-66	30	0.89
1987	-1	49	-34	17	0.95
1988	0	112	-72	35	0.94
1989	2	85	-70	30	0.90
1990	-1	96	-99	28	0.95
1991	1	88	-83	26	0.74
1992	-1	64	-70	24	0.85
1993	-1	50	-58	20	0.93
1994	0	69	-63	26	0.96
1995	0	101	-38	23	0.91
1996	1	45	-60	22	0.97
1997	-2	30	-44	19	0.96
1998	-1	51	-38	15	0.71
1999	4	60	-46	25	0.90
Mean of yearly residuals	0.08	75	-66	26	0.90

Table 12- Namakan Lake 2000 Rule Curves Model Outflow Statistics

Year	Mean Residual (m3/s)	Max Residual (m3/s)	Min Residual (m3/s)	Root Mean Squared Error (m3/s)	R ²
2000	0	57	-63	19	0.88
2001	0	71	-67	26	0.98
2002	0	58	-102	26	0.81
2003	-1	13	-34	9	0.95
2004	1	81	-45	19	0.95
2005	-1	109	-83	27	0.95
2006	1	151	-36	27	0.86
2007	1	36	-27	14	0.99
2008	0	87	-55	23	0.98
2009	1	50	-41	17	0.98
2010	0	88	-57	22	0.87
2011	0	126	-37	23	0.93
2012	1	61	-52	22	0.97
2013	-1	82	-83	26	0.96
2014	2	88	-78	27	0.98
Mean of Yearly Residuals	0.28	77	-57	22	0.94

Table 13 - Rainy Lake 1970 Rule Curves Model Outflow Statistics

Year	Mean Residual (m3/s)	Max Residual (m3/s)	Min Residual (m3/s)	Root Mean Squared Error (m3/s)	R ²
1970	0	284	-138	95	0.85
1971	1	201	-109	64	0.92
1972	-4	222	-153	76	0.54
1973	4	132	-97	44	0.88
1974	-3	274	-274	81	0.88
1975	-1	219	-92	57	0.81
1976	-1	217	-185	59	0.74
1977	4	159	-87	40	0.99
1978	-1	188	-170	74	0.82
1979	6	230	-182	66	0.91
1980	-1	71	-157	49	0.70
1981	-4	167	-110	50	0.79
1982	3	267	-219	88	0.72
1983	0	212	-91	54	0.67
1984	-1	143	-145	55	0.83
1985	1	267	-155	81	0.90
1986	-4	203	-138	72	0.83
1987	1	63	-140	44	0.65
1988	4	135	-137	51	0.95
1989	0	233	-169	76	0.83
1990	-4	185	-164	65	0.89
1991	2	124	-219	55	0.56
1992	0	233	-124	62	0.86
1993	-4	173	-213	67	0.82
1994	3	189	-100	54	0.87
1995	2	157	-104	47	0.86
1996	-1	243	-129	66	0.93
1997	-4	288	-106	63	0.81
1998	16	78	-27	26	0.71
1999	-16	174	-334	82	0.72
Mean of yearly residuals	-0.04	191	-149	62	0.81

Table 14 - Rainy Lake 2000 Rule Curves Model Outflow Statistics

Year	Mean Residual (m3/s)	Max Residual (m3/s)	Min Residual (m3/s)	Root Mean Squared Error (m3/s)	R ²
2000	2	88	-173	45	0.85
2001	0	244	-191	58	0.96
2002	-1	220	-163	54	0.97
2003	0	33	-21	12	0.93
2004	1	144	-83	52	0.89
2005	-1	159	-83	41	0.97
2006	-3	206	-171	47	0.86
2007	3	69	-127	41	0.97
2008	0	272	-112	60	0.96
2009	-2	129	-166	47	0.96
2010	0	154	-127	46	0.79
2011	2	206	-296	59	0.86
2012	2	125	-70	35	0.96
2013	-1	167	-133	52	0.95
2014	2	125	-87	38	0.99
Mean of Yearly Residuals	0.29	156	-134	46	0.92